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# Pooling mortality risk in Eurozone state pension liabilities: An application of a Bayesian coherent multi-population cohort-based mortality model

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ABSTRACT

We design a coherent cohort-based multi-population mortality model, calibrate it to national mortality rates in the Eurozone using Human Mortality Database data, and use it to project developments in national mortality across the Eurozone. Combining this model with a stylized model of social security pensions in each country allows us to calculate the pension mortality risk in these systems and estimate the benefits of pooling it across the Eurozone. We examine three risk pools, which are all actuarially fair, but differ in how undiversifiable risk is allocated across countries. The first naïve approach allocates undiversifiable risk in proportion to GDP, a second according to a CAPM-based measure of the undiversifiable risk each country contributes to the pool and a third ensures that the aggregate benefits of diversification are shared equitably across countries using a measure we adopt. In all cases, the benefits of risk pooling increase over time as mortality uncertainty accumulates, but fall over time as cross-country correlation increases due to the long-term dominance of the mortality trend, which by assumption is shared between countries. The peak benefit occurs around 2050, with an aggregate reduction in the standard deviation of pension expenditures of around 0.11% of GDP, or 3% of pension expenditure at the 99th percentile. We find that allocating undiversifiable risk proportional to GDP does not ensure an efficient allocation of undiversifiable risk across countries, given that different countries have markedly different pension mortality risk due to different pension system generosities as well as different mortality correlation with the Eurozone. Based on our results we propose a contract design that surmounts most of the moral hazard risks created by the pool, and suggest directions for future research.


## 1. Introduction

Despite recent reforms, European pension systems are, and are scheduled to remain, very costly. As shown in Table 1, the average cost of public pension systems across the Eurozone is currently around 10.8% of GDP per year, and projected to rise by 1.1% of GDP per year by 2040, in response to rapid population ageing.[2] As a consequence, the expected discounted present value of state pension promises already accrued to workers in the Eurozone – a form of public debt – is extremely high. At 258% of GDP in 2015,[3] the last year in which statistics are available, the size of this debt dwarfs both private-sector funded DB pension liabilities (17% of GDP in 2015 in the Eurozone) and standard measures of public debt (90.9% of GDP in 2015).[4] Because state pensions are, by definition, paid only until death, it stands to reason that the amount of longevity risk embedded within these promises is significant.

A great deal of academic work has examined how the mortality risk in private-sector insurance and pension liabilities can be

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1 The authors would like to thank discussants and participants at Longevity 15 conference in Washington, DC, the World Risk and Insurance Economics Congress (WRIEC 2020), an anonymous referee and the journal editors for useful comments and suggestions. All errors remain our own.

2 Table 1 also shows the projected development of the Old-Age Dependency Ratio (OADR, the number of people age 65 and over relative to the number between 20 and 64, from the 2019 Eurostat population projections (Eurostat Database, 2019)), and the size of public pension debt as a % of GDP, from the supplementary table on pensions, Eurostat. Projected pension expenditures are from reports of the Ageing Working Group (AWG) of the European Commission (2018).

3 This value and the value of private pension debt comes from the supplementary table on pensions prepared by Eurostat.

4 This is the value of government consolidated gross debt for general government, from Eurostat.

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**Table 1**
Measures of population ageing and pension expenditures, by country.

| Country | Panel A | | | | | | | Panel B | | | | | | | Panel C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Old-age dependency ratio (population 65 and over to population 20 to 64 years)‡ | | | | | | | Pension expenditures (% current GDP)* | | | | | | | SS pension liabilities (2015, % GDP)† |
| | 2015 | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | 2015 | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | |
| Austria | 29.8 | 30.8 | 39.8 | 48.0 | 51.3 | 54.7 | 55.8 | 13.5 | 13.5 | 14.1 | 14.5 | 14.2 | 14.3 | 13.9 | 303 |
| Belgium | 30.5 | 32.8 | 40.2 | 45.9 | 49.1 | 51.8 | 53.2 | 12.6 | 12.6 | 13.8 | 14.5 | 14.7 | 14.9 | 15.0 | 229 |
| Estonia | 30.9 | 34.1 | 40.7 | 45.7 | 53.3 | 61.5 | 59.3 | 7.8 | 7.8 | 7.2 | 7.1 | 7.1 | 6.9 | 6.4 | 246 |
| Finland | 34.4 | 39.4 | 46.6 | 48.3 | 52.1 | 57.9 | 62.2 | 13.8 | 13.8 | 14.8 | 13.9 | 13.2 | 13.5 | 13.9 | 301 |
| France | 32.3 | 36.9 | 44.5 | 51.6 | 54.6 | 56.0 | 56.8 | 15.0 | 15.0 | 15.4 | 15.1 | 13.8 | 12.5 | 11.8 | 316 |
| Germany | 34.6 | 36.4 | 45.8 | 52.3 | 52.8 | 54.3 | 54.6 | 10.3 | 10.3 | 11.5 | 12.0 | 12.2 | 12.5 | 12.5 | 226 |
| Greece | 35.1 | 38.2 | 45.6 | 57.2 | 68.1 | 67.4 | 65.3 | 13.4 | 13.4 | 12.0 | 12.9 | 12.5 | 11.5 | 10.6 | ¶288 |
| Ireland | 21.8 | 24.5 | 30.0 | 37.0 | 46.2 | 49.9 | 52.8 | 3.8 | 3.8 | 4.3 | 5.2 | 6.1 | 6.3 | 6.0 | 88 |
| Italy | 36.4 | 39.2 | 47.4 | 60.8 | 66.5 | 65.6 | 65.5 | 15.6 | 15.6 | 17.2 | 18.7 | 17.3 | 15.1 | 13.9 | 341 |
| Latvia | 31.7 | 34.9 | 45.3 | 53.4 | 61.8 | 69.7 | 63.7 | 6.8 | 6.8 | 6.2 | 6.3 | 6.1 | 5.6 | 4.7 | 172 |
| Lithuania | 30.8 | 33.1 | 44.9 | 55.6 | 61.2 | 68.3 | 66.2 | 7.0 | 7.0 | 7.1 | 7.0 | 6.5 | 6.0 | 5.2 | 206 |
| Luxembourg | 22.4 | 22.7 | 29.1 | 37.4 | 45.1 | 52.6 | 56.0 | 9.0 | 9.0 | 10.2 | 11.5 | 13.0 | 16.0 | 17.9 | ¶276 |
| Netherlands | 29.9 | 33.2 | 41.9 | 49.3 | 49.2 | 51.3 | 55.1 | 7.0 | 7.0 | 7.5 | 8.5 | 8.2 | 7.9 | 7.9 | 167 |
| Portugal | 33.8 | 37.6 | 46.7 | 58.8 | 68.8 | 68.0 | 67.3 | 13.6 | 13.6 | 14.3 | 14.7 | 13.7 | 12.0 | 11.4 | 206 |
| Slovakia | 21.4 | 26.4 | 35.6 | 42.5 | 55.8 | 66.3 | 63.3 | 8.3 | 8.3 | 7.6 | 7.8 | 8.8 | 9.9 | 9.8 | 283 |
| Slovenia | 28.5 | 33.7 | 43.0 | 50.5 | 59.6 | 61.8 | 58.9 | 11.0 | 11.0 | 12.0 | 14.2 | 15.6 | 15.2 | 14.9 | 313 |
| Spain | 30.0 | 32.3 | 40.4 | 53.2 | 64.5 | 64.2 | 62.5 | 12.3 | 12.3 | 12.6 | 13.9 | 13.9 | 11.4 | 10.7 | 258 |
| EUROZONE | 32.8 | 35.6 | 43.9 | 52.9 | 57.5 | 58.7 | 58.9 | 10.8 | 10.8 | 11.4 | 11.9 | 11.7 | 11.0 | 10.7 | 258 |

Note: ‡Data other than 2015 from Eurostat 2019 population projections (Eurostat Database, 2019), baseline variant. Data from 2015 from Eurostat. *Data other than 2015 from projections from country pension fiches prepared by the Ageing Working Group of the European Commission. 2015 data from pension fiches. †Data from Eurostat supplementary tables on pensions. ¶Luxembourg and Greece did not report SS pension liabilities to Eurostat. An appropriate productivity growth rate (0% p.a. for Luxembourg and 1% p.a. for Greece) was assumed and the values reported here are as estimated by the pension model described in the text.

transferred to facilitate better risk sharing. Yet despite the large size and extraordinary financial significance of state pensions in Europe, as far as we are aware, mortality risk transfer in public sector pensions has been soundly ignored in both the academic and the policy literature. We argue that the economic benefits of pooling longevity risk in state pension systems are potentially large, because by making pension expenditure more stable, the need for potentially disruptive and politically difficult pension reforms could be reduced. In this paper, we take a first look at this issue by focusing on the mortality risk in the national public pension systems of Eurozone countries.

We choose Eurozone countries for several reasons. Firstly, as already stated, European countries are ageing rapidly and typically have pension systems that are large relative to the size of their economies. Second, these countries share a common currency and a governance framework provided by the EU, meaning that mortality risk transfer is potentially much simpler than it might be between less closely integrated nations. Finally, there is already some cross-national risk transfer in the EU, notably through the reinsurance of national unemployment insurance systems. Mortality risk transfer is therefore only a small step away from current practice.

To explore the benefits of pooling the mortality risk in social security pensions in the Eurozone, we first build a coherent, multi-population cohort-based mortality forecasting model, and then calibrate it to the various countries of the Eurozone using data from the Human Mortality Database (HMD) (2019). We then use data from Eurostat and the AWG to build a highly stylized model of pension liabilities in each country.[5] The pension model calculates average pensions based on projections as they stood at the time of the most recent AWG reports (that is, 2018): so all reforms legislated before 2018 are included.[6] We then define pension mortality risk to be the change in pension expenditure if average pension amounts are equal to these projections, but mortality alone is different from what was expected. Using this approach, we are able to explore pension mortality risks in individual countries as well as how the pension mortality risk of each country fits into the pension mortality risk of the Eurozone as a whole. We then use these results to explore the potential benefits of pooling pension mortality risk across the Eurozone under various risk-pooling arrangements.

The academic literature shows some evidence of international convergence of mortality rates, but also notes that this convergence is far from complete. Wilson (2001) finds a global convergence in mortality levels by comparing the distributions of the global population by life expectancy over three different time periods, 1950–1955, 1975–1980, and 2000. White (2002) considers mortality data from 21 high-income countries over the period of 1955–1996, and finds similar results. The fact that convergence is not complete suggests that differences between country mortality rates could indeed provide some opportunities for further diversification between different countries.

While stochastic mortality modeling goes back to Lee and Carter (1992), stochastic multi-population modeling is rather newer. Li and Lee (2005) were the first to extend the Lee-Carter model to a multi-population setting. They considered the common part in mortality across populations in the model to ensure convergence. Li et al. (2015) propose a two-population stochastic mortality model and provide two-population generalizations for each of the 7 single-population models reviewed by Cairns et al. (2009). Kleinow (2015) introduces a model for the mortality rates of multiple populations. To build the proposed model he investigated to what extent a common age effect can be found among the mortality experiences of several countries and use a common principal component analysis to estimate a common age effect in an

[5] As discussed in a later section, our model is limited by data availability: we have mortality data for males and females for each country, but not state pension expenditure or pension liabilities. We have no data for either mortality or state pensions by socio-economic group. Although the mortality gap between men and women may be narrowing, many authors have pointed to widening disparities between the mortality of different socio-economic groups (e.g. Goldman and Orszag, 2014, for the US), and there may be widening differences in pension entitlements by socio-economic group, too.

[6] Note that national pension schemes are continually being altered. We ignore all reforms passed after the most recent AWG reports, and further assume that all reforms legislated up to 2018 will be implemented as planned. We thank a referee for pointing out that, for example, in Spain as of early 2021 legislated reforms have not yet been implemented as projected in the AWG reports of 2018.

age–period model for multiple populations. Enchev et al. (2017) review a number of multi-population mortality models: variations of the Li and Lee model, and the common-age-effect (CAE) model of Kleinow (2015). Li et al. (2017) propose a semi-coherent approach to relax the Li and Lee coherence assumption. Xu et al. (2018) propose a multi-cohort continuous-time affine mortality model and, along with an affine arbitrage-free term structure model, determine implied market prices of longevity risk in the BlackRock CoRI Retirement Indexes. Like Xu et al. (2018), we adopt a cohort-based approach, as this fits the nature of the liabilities best. However, we adopt a Bayesian approach to estimating the model pioneered by McCarthy (2021) and McCarthy and Wang (2021). This method uses a two-step approach to estimation that produces more stable estimates for recent cohorts, overcoming the cohort censoring problem that bedevils most cohort analysis. Coherence across multiple populations is incorporated using the coherence approach of Li and Lee (2005), although other approaches could easily be followed.

There is also a substantial literature on mortality hedging, but exclusively applied to private pension schemes. Coughlan et al. (2011) propose a framework for longevity basis risk analysis and hedge effectiveness and present a U.K. case study, which compares the population of assured lives from the Continuous Mortality Investigation with the England and Wales national population. Despite the different demographic profiles, the case study provides evidence of stable long-term relationships between the mortality experiences of the two populations. Cairns et al. (2014) use a case study of a pension plan wishing to hedge the longevity risk in its pension liabilities at a future date. They show how correlation and, therefore, hedge effectiveness can be broken down into contributions from a number of distinct types of risk factors: basis risk, recalibration risk, pension risk. Sherris et al. (2018) develop value-based longevity indexes for multiple cohorts in two different countries that take into account the major sources of risks impacting life insurance portfolios, mortality and interest rates. However, they do not apply their model to incomplete cohorts, seriously limiting its applicability to real-world portfolios.

We propose three very simple pooling mechanisms, all based on the mutualization of pension mortality risks across the Eurozone through a set of mortality swaps. All three are actuarially-fair, but differ in how systematic risk is allocated across countries. We choose to alter the payment leg of the mortality swaps to allocate this risk, and so also refer to the allocation method as premium risk adjustment where appropriate.

The first is a basic pool, where undiversifiable risk is allocated to countries in proportion to their GDP, so without any premium risk adjustment. We show that while this mechanism achieves a system-wide reduction in the standard deviation of pension expenditure, it does not allocate undiversifiable risk efficiently across different countries. We therefore also examine two different methods of risk-adjusting premiums in the pool.

Because we are examining national mortality risk, previous results estimating the price of mortality risk by calibrating mortality models to prices of mortality-linked securities arguably do not apply here. First, the amount of risk transferred is much larger than the amounts transferred in the private longevity-swap market. It is very unlikely that the price of aggregate mortality risk will be well approximated by the price of the few small and marginal transactions that have occurred in the private sector. Secondly, private-sector entities face structural constraints – related to solvency, capital requirements, and other regulations – that will undoubtedly alter the price at which they are willing to trade mortality risks. These factors will not influence the price of national mortality risk transfers. Finally, national mortality has much less idiosyncratic risk than individual pension plans and insurance companies. Investors may price this risk due to capital imperfections and for other reasons.

On these grounds, we choose to price mortality risk from first principles. Our first approach uses a version of the Capital Asset Pricing Model (CAPM) applied to pension expenditure that we develop, in which premiums are set based on the amount of undiversifiable pension mortality risk each country contributes to the pool. This achieves a very similar reduction in the standard deviation of aggregate risks to the basic risk pool, but allocates the undiversifiable risk across countries in a much more efficient way. Using the CAPM has some challenges, however. In particular, not all countries receive the same benefit of participating in the risk pool, and it is theoretically possible that some countries may be worse off after participating in the pool under this premium structure (although this does not happen in our dataset). We therefore develop a general framework for risk adjustment that is based on apportioning undiversifiable risk across countries in a way that shares the benefits of diversification equitably, according to a measure we choose, and illustrate how it may work using an example.

An important issue when valuing the benefits of insurance is the appropriate discount rate to use when discounting benefits and premiums. Because both non-risk-adjusted and risk-adjusted pools pay out the full aggregate premium received in each period, this issue does not arise in the aggregate case. In individual countries, however, it will. We defer to future work a discussion of the correct risk-adjusted discount rate to use for these types of risks,[7] and focus here on the reduction in the risk of pension expenditures within each country and across the Eurozone.

The paper proceeds as follows. The next section describes the mortality and pension data we use to calibrate the model. A section discusses the theory underlying the mortality model and the method used to fit it, the pension model used to approximate the pension liabilities of each country, and the risk-pooling mechanisms we design. A subsequent section presents our results, while the final section is a discussion and conclusion, where we also discuss some limitations of our approach and suggest avenues for future research.

## 2. Data

The data we use to fit the mortality and pension models is summarized in Tables 1 and 2.

[7] While the appropriate discount rate is moot at the aggregate level, at individual country level it becomes important because claims will almost certainly not equal premiums in any period. There is a broad discussion in the academic literature on the appropriate discount rate to use for long-run public-sector projects, going back to the Arrow-Lind theorem (Arrow and Lind, 1970). Note that because national pension systems are very large, the Arrow-Lind theorem arguably cannot apply to these liabilities, as it is based on the ability of the public sector to fully diversify project-specific risk. Most authors have therefore suggested that the appropriate discount rate for systematic risk should fall as the time horizon lengthens, owing to increased uncertainty and other factors. Some have even suggested that at very long horizons, the appropriate discount rate should be zero or very close to it. See Gollier (2002) and Gollier and Cherbonnier (2018) for a summary of these discussions. A further complication is that pension systems are transfer systems, so that although higher social security contributions likely reduce the consumption of working people, they raise the consumption of those already retired. A CCAPM-type approach would therefore price changes in social security contributions and benefits only to the extent that savings rates differed across cohorts, or that social security taxes induce dead-weight losses, leading to reductions in consumption that are arguably not fully contemporaneous with the increase in social security contributions. Barro (2006, 2009, 2013) show that the general problems with using the CCAPM to price risk noted by Mehra and Prescott (1985) and many others could possibly be surmounted by incorporating the risk of rare disasters into the standard model. Because of these complications, we defer exploration of this important issue to future work.

**Table 2**
Summary of data used for mortality and pension model.

| | Panel A‡ | | Panel B* | | | | | | Panel C* |
|---|---|---|---|---|---|---|---|---|---|
| | Mortality data from HMD | | Projected retirement ages (as legislated by 2018) | | | | | | Projected pension increases (as legislated by 2018) |
| | Earliest period | Latest period | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | |
| Austria | 1947 | 2017 | 63.6 | 63.6 | 64.5 | 65 | 65 | 65 | prices |
| Belgium | 1841 | 2015 | 65 | 67 | 67 | 67 | 67 | 67 | prices plus 1% p.a. |
| Estonia | 1959 | 2017 | 65 | 65.8 | 66.7 | 67.6 | 68.5 | 69.3 | 50% prices 50% wages |
| Finland | 1878 | 2015 | 63.8 | 65 | 65 | 65 | 65 | 65 | 50% prices 50% wages |
| France | 1816 | 2017 | 68 | 70 | 70 | 70 | 70 | 70 | 80% prices 20% wages |
| Germany | 1956 | 2017 | 67 | 67 | 67 | 67 | 67 | 67 | prices |
| Greece | 1981 | 2013 | 65.7 | 67 | 67 | 67 | 67 | 67 | wages less 0.5% p.a. |
| Ireland | 1950 | 2014 | 67 | 68.7 | 69.6 | 70.5 | 71.7 | 72.6 | prices less 0.5% p.a. |
| Italy | 1872 | 2014 | 66 | 68 | 68 | 68 | 68 | 68 | wages |
| Latvia | 1959 | 2017 | 67.1 | 67.9 | 68.8 | 69.6 | 70.4 | 71.1 | prices |
| Lithuania | 1959 | 2017 | 63.8 | 65 | 65 | 65 | 65 | 65 | 50% prices 50% wages |
| Luxembourg | 1960 | 2014 | 63.7 | 65 | 65 | 65 | 65 | 65 | wages less 1% p.a. |
| Netherlands | 1850 | 2016 | 65 | 65 | 65 | 65 | 65 | 65 | prices plus 1% p.a. |
| Portugal | 1940 | 2015 | 63 | 65 | 65 | 65 | 65 | 65 | 70% wages 30% prices |
| Slovakia | 1950 | 2017 | 66.7 | 68 | 69.3 | 70.5 | 71.5 | 72.5 | prices |
| Slovenia | 1983 | 2017 | 66.5 | 67.2 | 67.8 | 68.3 | 68.8 | 69.3 | prices less 0.5% p.a. |
| Spain | 1908 | 2016 | 62.9 | 64.5 | 65.8 | 67 | 68.1 | 68.8 | 0.25% p.a. |
| EUROZONE | 1956 | 2016 | | | | | | | |

NOTE: ‡ Unisex population mortality between ages 55 and 100 used for all countries. Cohort data for years of birth 1886 to 1933 were used to estimate prior; all data used to fit posterior. * Data from country pension fiches prepared by the Ageing Working Group of the European Commission.

### 2.1. Mortality data

Mortality data is taken from the Human Mortality Database (HMD) (2019). Data is collected on a period basis. We use 1x1 tables, and pool male and female data for each country by summing the number of deaths and exposures and calculating the aggregate mortality rate. Similarly, we also calculate aggregate mortality for the entire Eurozone by pooling the data across countries.[8] As we are focusing on old-age mortality, we only consider individuals between the ages of 55 and 100. As we only have Eurozone period data from 1956 to 2016, we only use data for cohorts with more than 30 years of data in this sample to estimate the prior, that is those born between 1886 and 1931, but use the data for all the cohorts shown in Panel A of Table 2 to estimate the posterior distribution (see the estimation section below for more details).

### 2.2. Pension data

Pension data is taken from two places. Panel B of Table 1 shows country-specific projections of pension expenditures as a percentage of GDP in 2015, in 2020, and in each subsequent decade up to the year 2070. These data were obtained from the country pension fiches published by the AWG. Panel C of Table 1 shows accrued social security pension liabilities, as a % of GDP in 2015, provided by Eurostat (2015). The data typically assumes a linear accrual of individual entitlements, and is discounted at a 5% p.a. nominal (3% real) rate of interest. Technical details underlying the calculations are presented in Eurostat (2011). Two countries (Greece and Luxembourg) did not submit data on accrued pension liabilities to Eurostat. For these countries, we estimate accrued SS pension liabilities by using an assumed labor productivity growth rate (0% p.a. for Luxembourg and 1% p.a. for Greece) and using the model described in section 3.2 below.

We also take from AWG pension fiches changes in retirement ages, and scheduled increases in pensions in payment for each country. The retirement age is taken as the simple average of the male and female retirement ages. These sets of data are shown in panels B and C of Table 2.

[8] We also pool West and East Germany data from HMD to obtain longer period data for Germany.

## 3. Theory and methods

In this section, we first introduce the coherent multi-population cohort-based model we use. The model has two pieces: a 'leading' model, which we use to model the entire European population, and a 'following' model for each country. We then discuss the pension model, and explain how we calibrate it to data. A final section presents the pooling mechanisms we discuss, as well as the pension risk measures that we use.

### 3.1. Coherent multi-population cohort-based mortality model

We broadly follow Li and Lee (2005) in what follows, making adjustments for the cohort-based nature of the model where appropriate.

We first model the mortality development of the pooled mortality data, which we use as the 'leading' model. To do this, we use the model of McCarthy and Wang (2021) and McCarthy (2021). What follows is a sketch of the approach, and readers are referred to those papers for more detail. We therefore have the following, based on the Gompertz (1825) law of mortality:

$$ln\left(m^{g}_{x,c}\right) = \beta^{g}_{0,c} + \beta^{g}_{1,c}x + \gamma_x + \varepsilon^{g}_{x,c}. \tag{1}$$

We model the changes in the parameter values $\beta^{g}_{0,c}$ and $\beta^{g}_{1,c}$, written $\beta^{g}_{c}$ using a first-order VAR in differences, so:

$$\Delta\beta^{g}_{c} - B\Delta\beta^{g}_{c-1} - A = \varepsilon^{g}_{c}. \tag{2}$$

We then posit, following Li and Lee (2005), that the parameters of each individual subpopulation, which we index by $j$, $j = 1...m$, have a similar model to (1), but with parameters that equal the group parameters, plus a disturbance term unique to each subpopulation, so:

$$ln\left(m^{j}_{x,c}\right) = (\beta^{g}_{0,c} + b^{j}_{0,c}) + (\beta^{g}_{1,c} + b^{j}_{1,c})x + \gamma^{j}_{x} + \varepsilon^{j}_{x,c}. \tag{3}$$

We then propose, again following Li and Lee (2005), that the values of $b^{j}_{0,c}$ and $b^{j}_{1,c}$, written $b^{j}_{c}$, follow an AR(1)

$$b^{j}_{c} = C^{j} + D^{j}b^{j}_{c-1} + \varepsilon^{j}_{c}. \tag{4}$$

For coherence, we require that the spherical radius of $D^j$ (that is, the norm of the largest eigenvalue) is less than 1. Then, over the long run, $b_c^j$ converges in expectation to:

$$b^j = C^j(I - D^j)^{-1}, \tag{5}$$

meaning that the mortality gap between country $j$ and the group will not diverge without limit. In this sense, Li and Lee (2005) define the mortality of the subpopulation and the group as being *coherent*, a definition that we follow (although in a cohort sense).

#### 3.1.1. Mortality model estimation

We now focus on the estimation problem. Full details of the approach taken can be found in McCarthy (2021) and McCarthy and Wang (2021). Following that paper, we use a Bayesian technique to deal with cohort censoring. We wish to obtain posterior estimates of $\beta_c^g$, and $b_c^j$, conditional on the grouped and individual mortality data and the meta-parameters of the prior distributions for $\Delta\beta_c^g$, $b_c^j$, $j = 1 \dots m$. These meta-parameters are $A, B, C^j$, $D^j$ ($j = 1 \dots m$), $\Sigma_{\varepsilon_c^g}$ and $\Sigma_{\varepsilon_c^j}$ ($j = 1 \dots m$).

*Stage 1: Estimation of $A$, $B$, $\Sigma_{\varepsilon_c^g}$*

In the first stage, we fit model (1) cohort-by-cohort to reasonably complete cohorts of the grouped data. We consider only cohorts with more than 30 years of data in the age range between 55 and 100 (this leaves cohorts born between 1886 and 1931, inclusive). Call these values $\hat{\beta}_c^g$. We then use these estimates of $\beta_c^g$ to estimate the parameters of (2) (which are $A$, $B$ and the variance-covariance matrix of $\varepsilon_c$, $\Sigma_{\varepsilon_c}$) using least squares. Call these values $\hat{A}$ $\hat{B}$ and $\hat{\Sigma}_{\varepsilon_c}$.

*Stage 2: Bayesian estimation of $\beta_c^g$ for all cohorts (including incomplete cohorts)*

We perform this estimation using Bayesian maximum *a posteriori* (BMAP) estimation, which improve the stability of estimates for recent cohorts over those provided by, say, maximum likelihood or least squares. BMAP estimation chooses those parameter values that lie at the mode of the posterior distribution of those parameters, conditional on the prior distribution and the likelihood function of the data. Bayes' Theorem gives the posterior as, where $\Omega$ is the set of meta-parameters of the prior distributions:

$$\log p_2(\beta|\Omega, \{m_{x,c}^g\}) = K + \log \ell(\{m_{x,c}^g\}|\beta) + \log p_1(\Delta\beta|\Omega) \tag{6}$$

We cluster standard errors by age to allow age random effects to be estimated. We define $\hat{\beta}_c^g$ as the mode of the posterior distribution $\log p_2(\beta|\Omega, \{m_{x,c}^g\})$. When estimating the posterior, we use all data available, that is, for cohorts born between 1856 and 1961, therefore including incomplete cohorts.

*Stage 3: Estimation of $C^j$, $D^j$ and $\Sigma_{\varepsilon_c^j}$ ($j = 1 \dots m$)*

For each sub-population, we pool male and female data and then estimate a mortality model for reasonably complete cohorts analogous to (1). This procedure gives values we call $\hat{\beta}_c^j$. We then estimate values of $b_c^j$ for these cohorts as:

$$\hat{b}_c^j = \hat{\beta}_c^j - \hat{\beta}_c^g. \tag{7}$$

We then use these estimates $\hat{b}_c^j$ to estimate the meta-parameters of (4) for each subpopulation again using least squares. We call these estimates $\hat{C}^j$, $\hat{D}^j$ and $\hat{\Sigma}_{\varepsilon_c^j}$ ($j = 1 \dots m$). Once again, we only use reasonably complete cohorts for which we have both country and Eurozone data to estimate this prior, that is, cohorts born between 1886 and 1931, inclusive.

*Stage 4: Bayesian estimation of $b_c^j$ for all cohorts (including incomplete cohorts)*

We calculate the posterior distribution using Bayes Theorem as a constant plus the sum of the logged likelihood function of the data conditional on $\beta$ and $b_c^j$, and the logged joint density function of the prior for $b_c^j$. We treat $\beta_c^g$ as fixed in this step:

$$\log p_2(b_c^j|\Omega, \{m_{x,c}^j\}) = K + \log \ell(\{m_{x,c}^j\}|\beta, b_c^j) + \log p_1(b_c^j|\Omega), \tag{8}$$

where (written in matrix form):

$$\begin{aligned} p_1(b^j|\Omega) = \Phi(&(I_{n-1} - \hat{D}^j L)^{-1}\hat{C}^j, \\ &(I_{n-1} - \hat{D}^j L)^{-1}(I_{n-1} \otimes \hat{\Sigma}_{\varepsilon_c^j})(I_{n-1} - \hat{D}^j L)^{-1T}), \end{aligned} \tag{9}$$

where $I_{n-1}$ is an identity matrix of size $n-1$, $L$ is the lag operator and $\otimes$ is the Kronecker product. We then calculate the BMAP estimates of $b_c^j$ for all cohorts as those values that maximize the posterior distribution $\log p_2(b_c^j|\Omega, \{m_{x,c}^j\})$. Once again, age random effects are clustered.

This estimation procedure – starting with the general population, then estimating deviations for sub-groups off residuals – could be continued through various levels of hierarchy, ostensibly without end. Here, we use it only for the Eurozone and each country, and so have two levels of hierarchy. We could add regional variations in mortality (above or below country level), as well as mortality by sex or socio-economic group.

When estimating the posterior, as before we use all data for each country, including earlier data and incomplete cohorts. We note that earlier data (before WWII) in the HMD is often unreliable. This has little effect on our results – partly because the cohort-based approach means that cohort estimates are averages over many time periods, and partly because our projections are based on the estimates for the latest cohorts only, upon which earlier data unreliability has little or no influence.

### 3.2. Country pension model

Pension systems in Europe fall into three main types – pay-as-you-go earnings-related or flat-rate defined benefit (DB) systems, points-based systems, and notional defined contribution (NDC) systems. The system types differ according to the method used to determine pension amounts and increases in pensions while in payment. Eligibility conditions for retirement – often a combination of age and length of contributory period – are usually specified exogenously in all system types. Although the systems are designed to spread the cost of small fluctuations in longevity around the population as a whole – and do so in different ways – the extent of population ageing is way larger than can be managed under 'normal' operations of these systems. Most countries have therefore adopted additional reforms to reduce future pension costs – increasing retirement ages, reducing indexation of pensions-in-payment, reducing initial pension amounts – or have increased contributions, usually a combination of these. Appendix B describes the pension systems of the different countries as they were included in our model (a very brief summary: only public tier-one pensions are modeled, including all pension reforms legislated before 2018 and their long-run effect on pension expenditures).

Creating a full model of pension expenditure in each country would be extremely complicated to implement, and far beyond the scope of the current paper. Instead, we model a highly simplified version of the pension system in each country by modeling an average pension amount, adjusted for changes in productivity, and projected changes in retirement ages and pension-in-payment indexation, in each future year in each country. The model estimates average individual expectations of pension amounts given retirement ages under current law. We use the model results in combination with different population structures caused by different longevity outcomes to measure pension mortality risk for all pension system types.

Lacking data on pension expenditures by males and females, we pool male and females into a single model, despite significant differences between the pension entitlements of the two groups. We also abstract from socio-economic differences in pensions (and associated differences in longevity development) for the same reason. We calibrate the model by choosing a constant productivity growth rate and average pension amounts in each country in each future year to match the data in panels B and C of Table 1. Importantly the manner in which pension amounts and increases are set (i.e. the type of pension system (DB, points, NDC)) does not therefore affect the validity of the model in any way.[9]

We now describe the pension model in more detail. We let the retirement age of individuals born in year $c$ in country $j$ be $RA_{j,c}$,[10] assume average labor market entry at age 24,[11] real growth in wages of $g_j$ p.a. set equal to the (constant) rate of labor productivity growth in country $j$, inflation of $i$ p.a., constant across all countries (as they share a common currency), a real discount rate $r$ of 3% p.a., to match the Eurostat assumptions, and that pensions in payment increase (in real terms) at rate $v_{t,j}$ in year $t$ (shown in panel C of Table 2). The average pension benefit amount factor in country $j$ in year $t$ after adjusting for productivity gains and pension-in-payment increases we call $AF_{j,t}$.

Then, the average amount of the pension paid to an individual born in year $c$ in country $j$ in year $t$ can be calculated as:

$$p_{j,c,t} = \underbrace{AF_{j,t}}_{\substack{\text{Average}\\\text{pension}\\\text{amount}\\\text{factor}}} \times \underbrace{(1+g_j+i)^{RA_{j,c}-(2015-c)}}_{\substack{\text{Adjustment for nominal wage growth}\\\text{until retirement}}} \times \underbrace{\prod_{i=c+RA_{j,c}+1}^{t} (1+v_{j,i}+i)}_{\substack{\text{Adjustment for nominal pension}\\\text{increases after retirement}}} \tag{10}$$

if $t-c > RA_{j,c}$, and 0 otherwise.

We calculate the present value of all such future payments in country $j$ in the year 2015, that have accrued to individuals by the year 2015, which we call $AL_{j,2015}$. In line with Box 10 of Eurostat (2011), we assume a linear accrual of pension benefits between age 24 and retirement age. $N_{j,c,t}$ is the number of individuals in country $j$ born in year $c$ who are alive at time $t$.

$$AL_{j,2015} = \sum_c \underbrace{\max[\min[\frac{(2015-24-c)}{RA_{j,c}-24}, 1], 0]}_{\substack{\text{Proportion of pension payments}\\\text{that has accrued by 2015,}\\\text{assuming entry age of 24}}} \times \sum_t \underbrace{N_{j,c,t}}_{\substack{\text{No of}\\\text{people}}} \underbrace{p_{j,c,t}}_{\substack{\text{Average}\\\text{pension}\\\text{amount}}} \underbrace{(1+r+i)^{2015-t}}_{\text{Discount factor}} \tag{11}$$

Similarly, we calculate the total pension payments payable in year t in country j as:

$$P_{j,t} = \sum_{c \text{ st } t-c>RA_{j,c}} \underbrace{N_{j,c,t}}_{\substack{\text{No of}\\\text{people}}} \underbrace{p_{j,c,t}}_{\substack{\text{Average}\\\text{pension}\\\text{amount}}} \tag{12}$$

We choose values of $\{AF_{j,t}\}$ and $g_j$ to set $AL_{j,2015}$ from equation (11) equal to the accrued pension liability in panel C of Table 1 for each country, and to set $P_{j,t}$ from equation (12) equal to the corresponding percentage of GDP for each country in each year in panel B of Table 1. For years between those points, we interpolate linearly. We assume that per capita GDP grows at the rate of nominal labor productivity (so aggregate GDP grows at the rate of nominal labor productivity plus population growth).

### 3.3. Forecasting pension expenditures as a percentage of GDP

Once the mortality model and the pension model have been calibrated to the data, we can then forecast pension expenditures as a percentage of GDP under different scenarios of longevity progression. The projection methodology is similar to that followed

[9] Strictly speaking, NDC plans, like cash balance plans, are defined benefit plans where the benefit is defined as the income that can be bought at retirement under plan rules using the accumulated value in the notional account. They can – and we argue, should – therefore be valued prospectively in exactly the same way as any other DB plan, which is the approach we adopt here. Note that this prospective approach will give an accrued liability equal to the balance of the notional accounts only under quite special circumstances (where the accumulation rate in the notional accounts equals the discount rate and annuity conversion at retirement happens at actuarially fair rates). To see why this should not be not a source of concern, imagine two different notional account systems. The first sets the accumulation rate of the notional accounts equal to zero. The second sets the accumulation rate equal to nominal wage growth plus 5% p.a. Clearly, the second system will produce much larger pensions than the first for a given notional contribution rate and current notional account balances (assuming that annuity conversions happen at actuarially-fair rates at retirement in both). For this reason, setting the liability in both systems equal to the notional account value would therefore be incorrect, provided we are valuing the systems on a going-concern basis. Countries with notional account accumulation rates higher (lower) than the valuation discount rate will have true liabilities higher (lower) than the value of the notional accounts (reflecting the higher (lower) pensions provided by the system). In fact, the notional account balances are not even an estimate of the discontinuance liability, which, strictly speaking is again the present value of the pensions to be paid out under the terms of the discontinuance. If we made the assumption that the system could be shut down instantly (itself somewhat dubious), then the notional account balances would be an estimate of the discontinuance liability only if the terms of the discontinuance set the accumulation rate of the (frozen) notional accounts equal to the discount rate used in the calculation (and, again, annuity conversion was actuarially-fair), or if the discountenance terms paid out the notional account balances in cash. We follow a similar approach with points-based systems: calculate the average pension paid by the system in each future year, adjusted for growth, changes in retirement ages and pension increases; multiply the resulting average pension for each cohort by the portion of working life that has been completed by each cohort; discount this stream of pension payments using mortality and interest at the Eurostat-recommended rate, and sum up over all cohorts to get the total accrued liability. The 41-year assumption will make little difference to the final calibration, for the reason described in footnote 11 below.

[10] We first analyze retirement age by 10-year interval for all countries based on the country pension fiches of the Ageing Working Group of the European Commission (AWG). We interpolate these ages for all years and round to integers. Then we calculate the inferred cohort of these individuals and get $RA_{j,c}$.

[11] Assuming an entry of 24 allows the accrued liability to be calculated (i.e. Panel C in Table 1). Because retirement ages are usually around 65, this means that we are assuming around a 41-year working life (longer if retirement ages increase beyond 65). The pension model is based on (endogenous) estimates of the actual average pensions paid out, adjusted for productivity growth and changes in retirement ages. The assumption of linear accrual over a 41-year working life is only used to estimate the aggregate accrued pension liabilities of the system to current members of the population in respect of these average pensions. The assumption is therefore not that every individual has a complete working life; it is simply that the accrued liability of the population as a whole equals the liability that the system would have if every individual had accrued the portion of the final pension proportional to the portion of their 41-year working life that has actually been completed. This is similar to, but even weaker than assuming that that non-participation in the pension system (e.g. due to unemployment, full-time study, ill-health, career gaps or early retirement) is uncorrelated with age. Replacing the 41-year assumption with another number will therefore have little, if any, effect on our results.

for the US by McCarthy (2021). We first use the BMAP estimates of $\beta_c^g$ and $b_c^j$ to re-estimate parameters of their underlying processes – which we call $A, B, C^j, D^j$. We then do 10,000 runs of the following:

(1) We draw a sample from the posterior distribution for $\beta_c^g$ and $b_c^j$ ($j = 1...17$). We use these parameter values to re-estimate the residuals of the fitted model to obtain a set of period and age effects.
(2) We fit an AR(1) model to the estimated period effects, and project these forward by country. Note that we assume zero correlation between the period effects in each country. This likely understates the degree of correlation between the mortality rates in different countries at short time horizons.
(3) We project forward the values of $\beta_c^g$ and $b_c^j$ ($j = 1 \ldots 17$), using the values of $A, B, C^i, D^i$.
(4) We use the projected values of $\beta_c^g$ and $b_c^j$ ($j = 1 \ldots 17$), the period effects, and the age effects to calculate a set of mortality rates.
(5) We use the current population as per Eurostat in 2015, and migration and fertility assumptions from the 2019 Eurostat population projections (Eurostat Database, 2019) and these mortality rates to project the population forward.
(6) We then use the country pension model, and fitted values of $\{AF_{j,t}\}$ and $g_j$ for each country to calculate pension expenditures in each year in each country in each population scenario, as well as for the Eurozone as a whole.

### 3.4. Pension mortality pooling mechanisms

As before, we define $P_{j,t}$ to be the total amount of pension payments, in Euros, to pensioners in country $j$ in year $t$. We define the total pension payments in year t in the Eurozone as

$$P_{g,t} = \sum_j P_{j,t}, \tag{13}$$

the proportion of GDP spent on pension payments in country $j$ in year $t$ as

$$P^*_{j,t} = P_{j,t}/GDP_{j,t}, \tag{14}$$

where $GDP_{j,t}$ is the projected GDP of country $j$ in year $t$. The proportion of Eurozone GDP spent on pension payments as in year $t$ is then:

$$P^*_{g,t} = \sum_j P_{j,t} / \sum_j GDP_{j,t}. \tag{15}$$

We indicate means of each quantity with bars, so $\bar{P}_{j,t}$ is the mean projected pension expenditure of country $j$ in year $t$.

#### 3.4.1. Risk measurement

Before we discuss the risk pooling mechanism itself, it is convenient to discuss how we measure the benefits of participation. For each country, we examine the reduction in the risk of pension expenditure as a result of participating in the risk pool. We could use any measure of risk; for analytical convenience[12] we choose the reduction in the standard deviation of pension payments.

$$RR^{Sk}_{j,t} = sd(P^{Sk}_{j,t}) - sd(P_{j,t}), \tag{16}$$

where $sd(.)$ is the standard deviation of the random variable inside the parentheses, $P^{Sk}_{j,t}$ is the pension expenditure of country $j$ in year $t$ after participating in risk pool arrangement $k$, one of the three presented below.

Eurozone-wide benefits to diversification can be calculated as

$$\mathrm{A}(Sk) : \frac{\left[\sum_j RR^{Sk}_{j,t}\right]}{GDP_{g.t}}, \tag{17}$$

and country-specific benefits as

$$\mathrm{B}(Sk) : \frac{RR^{Sk}_{j,t}}{GDP_{j.t}}. \tag{18}$$

Both A and B are expressed as a proportion of GDP.

#### 3.4.2. Mortality risk pool without risk adjustment

We assume that all countries participate in a swap-type arrangement with a centralized mortality pool where they swap out their own pension mortality risk (defined as the difference between their actual and expected projected pension expenditures, where the actual pension expenditures differ from the expected only because of mortality changes, not other sources of risk) and in exchange accept the mortality risk of the entire Eurozone (defined as the difference between the actual and expected pension expenditures of the Eurozone).

The payments in year $t$ under this pooling agreement would then be as follows:

Country $j$ pays $(P^*_{g,t} - \bar{P}^*_{g,t}) \times GDP_{j,t}$ into the centralized pool
Country $j$ receives $P_{j,t} - \bar{P}_{j,t}$ from the centralized pool

Note that under this arrangement, all premiums are actuarially fair, since $E[P^*_{g,t}] = \bar{P}^*_{g,t}$ and $E[P_{j,t}] = \bar{P}_{j,t}$ by definition, and that the undiversifiable risk of the pool is shared across countries simply in proportion to their GDP, so the relative contribution of each country to the systematic risk of the pool, or the different benefits they obtain from participating, are ignored.

The total payments made by the centralized pool in year $t$ would then be:

$$\sum_j P_{j,t} - \bar{P}_{j,t} = P_{g,t} - \bar{P}_{g,t}, \tag{19}$$

while the total payments received would equal:

$$\begin{aligned} &\sum_j (P^*_{g,t} - \bar{P}^*_{g,t}) \times GDP_{j,t} \\ &= (P^*_{g,t} - \bar{P}^*_{g,t}) \sum_j GDP_{j,t} = (P^*_{g,t} - \bar{P}^*_{g,t}) GDP_{g,t} \\ &= P_{g,t} - \bar{P}_{g,t}. \end{aligned} \tag{20}$$

The pool therefore balances payments received with payments made in aggregate in each year.

Each country then makes pension payments promised to its retirees under its pension rules, pays what is due under the pooling agreement and receives the proceeds. So the pension expenditure of country $j$ at time $t$ after all payments and receipts from the common pool are:

$$\begin{aligned} P^{S1}_{j,t} &= P_{j,t} + (P^*_{g,t} - \bar{P}^*_{g,t}) \times GDP_{j,t} - (P_{j,t} - \bar{P}_{j,t}) \\ &= \bar{P}_{j,t} + (P^*_{g,t} - \bar{P}^*_{g,t}) \times GDP_{j,t}, \end{aligned} \tag{21}$$

their expected payments plus their share of actual Eurozone mortality risk. Without the swap agreement, each country pays simply $P_{j,t}$. Note that $\mathrm{var}(P^{*,S1}_{i,t}) = \mathrm{var}(P^{*,S1}_{j,t}) \forall i, j$, so each country has the same variance of pension expenditure as a percentage of GDP after participating in the pool.

[12] Standard deviation is a convenient measure because it has the same units as the original random variable and hence is relatively easy to interpret. In our case it is also analytically convenient because it permits closed-form solutions to be derived for risk-adjustment of premiums, shown in the text below.

### 3.4.3. Mortality risk pool with risk adjustment

One difficulty with the pure swap agreement as defined above is that premiums are not risk adjusted: undiversifiable risk is shared across countries based only on their GDP. But countries with more generous pension systems (where a given change in mortality rates has a larger effect on pension expenditures as a fraction of GDP) would then add more risk to the aggregate risk pool than countries with relatively less generous pension systems as a fraction of GDP, as would countries where the correlation between the country's mortality and the Eurozone's mortality is higher than average. Participation in this risk pool will therefore create potentially inequitable and therefore undesirable transfers between countries, leading to challenges on the grounds of fairness. A different allocation of undiversifiable risk across countries could help resolve this challenge. However, there is no uniquely best way to do this. We first examine an allocation scheme based on ensuring that each country bears the share of the undiversifiable risk it adds to the pool, based on the CAPM. We then illustrate a different scheme that is designed to allocate the undiversifiable risk between countries in a way that spreads the benefits of diversification equitably using a chosen measure.

*3.4.3.1. CAPM-based risk allocation scheme* As before, country $j$ receives $P_{j,t}-\bar{P}_{j,t}$ from the risk pool. However, assume that country $j$ pays $\beta^P_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t}$, where $\beta^P_{j,t}$ is a factor representing the aggregate risk they contribute to the pool. We note that under this premium structure, premiums are actuarially fair for any choice of $\beta^P_{j,t}$, since $E[P^*_{g,t}]=\bar{P}^*_{g,t}$ by definition.

Each country's pension expenditure after all payments to and from the mortality risk pool, is now written as:

$$\begin{aligned}P^{S2}_{j,t}&=P_{j,t}+\beta^P_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t}-(P_{j,t}-\bar{P}_{j,t})\\&=\bar{P}_{j,t}+\beta^P_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t},\end{aligned}$$

and as a fraction of GDP, as:

$$P^{*,S2}_{j,t}=\bar{P}^*_{j,t}+\beta^P_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t}).$$

The total undiversifiable mortality risk in the Eurozone is $\mathrm{var}(P_{g,t}-\bar{P}_{g,t})$. We can split this into the contribution of each country to the pool by noting that

$$\begin{aligned}\mathrm{var}(P_{g,t}-\bar{P}_{g,t})&=\mathrm{cov}(P_{g,t}-\bar{P}_{g,t},P_{g,t}-\bar{P}_{g,t})\\&=\sum_j\mathrm{cov}(P_{j,t}-\bar{P}_{j,t},P_{g,t}-\bar{P}_{g,t}).\end{aligned}$$

A little algebraic manipulation shows that setting

$$\beta^P_{j,t}=\frac{\mathrm{cov}(P^*_{g,t},P^*_{j,t})}{\mathrm{var}(P^*_{g,t})},\tag{22}$$

ensures that each country pays a premium equal to its contribution to the undiversifiable mortality risk in the pool, and therefore also that the total payments into and out of the risk pool are balanced each year, as we verify below:

$$\begin{aligned}&\sum_j\beta^P_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t}\\&=\sum_j\frac{\mathrm{cov}(P^*_{j,t},P^*_{g,t})}{\mathrm{var}(P^*_{g,t})}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t}\\&=\frac{(P^*_{g,t}-\bar{P}^*_{g,t})}{\mathrm{var}(P^*_{g,t})}\sum_j\mathrm{cov}(P^*_{j,t},P^*_{g,t})GDP_{j,t}\\&=\frac{(P^*_{g,t}-\bar{P}^*_{g,t})}{\mathrm{var}(P^*_{g,t})}\sum_j\mathrm{cov}\Big(\frac{P_{j,t}}{GDP_{j,t}},\frac{P_{g,t}}{GDP_{g,t}}\Big)GDP_{j,t}\\&=\frac{(P^*_{g,t}-\bar{P}^*_{g,t})}{\mathrm{var}(P^*_{g,t})GDP_{g,t}}\sum_j\mathrm{cov}(P_{j,t},P_{g,t})=\frac{(P^*_{g,t}-\bar{P}^*_{g,t})}{\mathrm{var}(P^*_{g,t})GDP_{g,t}}\mathrm{var}(P_{g,t})\\&=\frac{(P^*_{g,t}-\bar{P}^*_{g,t})}{\frac{\mathrm{var}(P_{g,t})}{GDP_{g,t}}}\mathrm{var}(P_{g,t})=(P^*_{g,t}-\bar{P}^*_{g,t})GDP_{g,t}=(P_{g,t}-\bar{P}_{g,t}).\end{aligned}\tag{23}$$

We call this the Capital Asset Pricing Model (CAPM) risk adjustment scheme. We note that this choice of $\beta^P_{j,t}$ can also be obtained by choosing that value of $\beta^P_{j,t}$ which minimizes the variance of the change in the country's pension expenditures as a result of participating in the pool, that is $\arg\min_\beta \mathrm{var}(P^{S2,*}_{j,t}-P^*_{j,t})=\beta^P_{j,t}$. This is consistent with the CAPM as the optimal approach of a mean-variance investor. But it is not at all obvious that countries should have mean-variance preferences.

One potentially undesirable feature of this allocation scheme is that although all countries pay premiums that fairly capture their contribution of aggregate risk to the pool, all countries will not enjoy the same benefits of participation, however these are defined. In fact, it is at least conceptually possible that a country may still actually be worse off after participating than it would have been on its own (although this does not happen in our sample). For instance, a country with zero variance in pension expenditures will pay no risk premium, but, after participating in the pool obtain a variance of pension expenditures equal to the Eurozone-wide variance, higher than its own variance.

*3.4.3.2. Equitable-benefit risk allocation scheme* This suggests a possible alternative way of allocating the aggregate risk: choose a sensible metric for the benefit of being in the risk pool relative to staying out of it, and then set premiums so as to allocate these benefits across different countries in an equitable way, subject to the overall constraint that the risk pool is self-financing in each year.

As a risk measure, we have chosen to use the reduction in standard deviation of pension expenditures as a proportion of GDP in year $t$ as a result of participating in the risk pool, described in section 3.4.1 above. Various ways of apportioning the total reduction in standard deviation across countries are possible. One choice might be to set the absolute reduction in the standard deviation of pension expenditures as a proportion of GDP to be equal for each country. Appendix C contains the mathematical derivation of such a rule. Unfortunately, in our sample this allocation method fails in earlier years (before 2035) because the standard deviation of the pension expenditure of some countries is too low to permit an equal reduction across all countries.

For each $t$, we therefore choose a set of $\{\beta^E_{j,t}\}_{j=1..N}$ to set the *proportional* reduction in standard deviation of pension expenditures as a result of participating in the risk pool equal for each country in that year and to ensure that payments into the risk pool in year $t$ are equal to payments out of the risk pool in that year.[13]

We first calculate

$$\begin{aligned}P^{S3}_{j,t}&=P_{j,t}+\beta^E_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t}-(P_{j,t}-\bar{P}_{j,t})\\&=\bar{P}_{j,t}+\beta^E_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t})\times GDP_{j,t},\end{aligned}$$

and as a fraction of GDP, as:

$$P^{*,S3}_{j,t}=\bar{P}^*_{j,t}+\beta^E_{j,t}(P^*_{g,t}-\bar{P}^*_{g,t}).$$

[13] Because betas are year-specific, a blended approach is possible. One option might be to follow the proportional reduction in the earlier years and the absolute reduction once this becomes feasible in the later years.

**Table 3**
Fitted pension model parameters.

| Country | Implied real productivity growth rate, $g$ | Pension adjustment factors (€'000's) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 2015 | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 |
| Austria | 0.0% | 27.742 | 26.881 | 23.322 | 21.362 | 20.944 | 20.175 | 19.305 |
| Belgium | −1.9% | 18.442 | 17.331 | 17.845 | 16.642 | 16.187 | 15.924 | 15.484 |
| Estonia | 3.4% | 7.204 | 6.731 | 5.828 | 5.185 | 4.696 | 4.188 | 4.128 |
| Finland | 0.2% | 34.296 | 29.221 | 29.658 | 25.540 | 24.178 | 22.967 | 21.725 |
| France | −0.0% | 31.052 | 27.469 | 23.809 | 20.862 | 18.594 | 17.036 | 15.974 |
| Germany | −0.2% | 20.344 | 19.664 | 20.147 | 18.180 | 18.683 | 18.764 | 18.593 |
| ‡Greece | 1.0% | 13.497 | 12.621 | 11.050 | 10.527 | 8.755 | 8.721 | 8.983 |
| Ireland | −1.3% | 17.783 | 15.808 | 15.483 | 15.303 | 14.749 | 13.832 | 12.479 |
| Italy | 0.2% | 22.379 | 20.774 | 20.809 | 19.481 | 16.849 | 14.901 | 14.819 |
| Latvia | 2.8% | 4.619 | 4.357 | 3.543 | 3.299 | 3.034 | 2.634 | 2.448 |
| Lithuania | 3.6% | 4.977 | 4.629 | 3.838 | 3.343 | 3.047 | 2.627 | 2.348 |
| †Luxembourg | 0.0% | 52.629 | 51.126 | 49.370 | 47.354 | 47.221 | 51.581 | 53.128 |
| Netherlands | 0.2% | 19.038 | 16.995 | 15.858 | 16.229 | 16.221 | 17.493 | 16.398 |
| Portugal | −2.3% | 10.207 | 9.269 | 8.483 | 7.668 | 6.161 | 5.377 | 5.112 |
| Slovakia | 3.6% | 9.855 | 8.415 | 7.013 | 7.269 | 6.936 | 7.268 | 7.773 |
| Slovenia | 1.4% | 12.210 | 10.726 | 9.554 | 9.987 | 10.007 | 9.777 | 10.145 |
| Spain | −0.1% | 18.347 | 17.272 | 15.416 | 13.752 | 12.513 | 10.729 | 10.204 |

NOTE: Eurostat reports pension liabilities calculated using a discount rate of 3% p.a. real. A price inflation rate of 2% p.a. was used. Growth rates and pension adjustment factors were adjusted by country to ensure that the pension model replicated the PV of SS pension liabilities reported by Eurostat and the annual pension expenditure as a percentage of GDP reported by the AWG. The adjustment factors reflect the generosity of the pension system in each country as well as parametric reforms implemented by 2016. † Luxembourg did not report a present value of pension liabilities. Hence a productivity growth rate of 0% p.a. was assumed and the pension liabilities reported were as calculated by the pension model. ‡ Greece did not report a present value of pension liabilities. Hence a productivity growth rate of 1% p.a. was assumed and the pension liabilities reported were as calculated by the pension model.

Note again that premiums are actuarially-fair for any choice of $\beta^E_{j,t}$. We derive $\beta^E_{j,t}$ as follows, where the proportional reduction in the country's standard deviation of pension risk equals $1-\alpha$:

$$\begin{aligned} sd(P^*_{j,t}) - sd(P^{*,S3}_{j,t}) &= sd(P^*_{j,t}) - \beta^E_{j,t} sd(P^*_{g,t}) \\ &= \alpha sd(P^*_{j,t}) \; \forall j = 1...N \end{aligned}$$

$$1 - \beta^E_{j,t}\frac{sd(P^*_{g,t})}{sd(P^*_{j,t})} = \alpha \tag{24}$$

$$\beta^E_{j,t} = (1-\alpha)\frac{sd(P^*_{j,t})}{sd(P^*_{g,t})}.$$

Now, the self-financing condition requires that

$$\sum_j \beta^E_{j,t} GDP_{j,t} = GDP_{g,t}.$$

Substituting the equation for $\beta^E_{j,t}$ into this and re-arranging gives:

$$\alpha = 1 - \frac{GDP_{g,t}}{\sum_j \frac{sd(P^*_{j,t})}{sd(P^*_{g,t})} GDP_{j,t}}.$$

Resubstituting back into the definition of $\beta^E_{j,t}$ gives

$$\beta^E_{j,t} = \frac{\frac{sd(P^*_{j,t})}{sd(P^*_{g,t})} GDP_{g,t}}{\sum_j \frac{sd(P^*_{j,t})}{sd(P^*_{g,t})} GDP_{j,t}}. \tag{25}$$

Since there is an infinite choice of objective functions, there is an infinite choice of different possible methods of risk allocation using this general approach. The choice of the 'best' option depends on the preference functions of the different countries and the political economy of the risk-sharing arrangement. Beyond noting that countries with mean-variance preferences will choose the CAPM-based approach discussed above, we leave this issue to future research.

## 4. Results

Table 3 shows the fitted pension model parameters.

There is a clear correlation between per capita GDP and the implied real productivity growth rates. In wealthier countries, with the exceptions of Ireland, Belgium and Portugal, projected productivity growth is close to 0% p.a., while poorer countries (notably those in the ex-Warsaw pact) have projected growth rates of around 3% p.a. Belgium, Ireland and Portugal have negative implied productivity growth rates, indicating that the pension liabilities reported are lower than the expenditure projections reported to the AWG would suggest.

Based on these slightly anomalous results, we debated whether to exclude calibration to the Eurostat (2015) supplementary table on pensions entirely. However, given that the large dispersion in implied productivity growth rates between the different countries (nearly 6% p.a. between the highest and the lowest) implies significant changes in the relative sizes of these countries' economies – and hence their pension systems – over the projection period, we felt that this should be left in the model to incorporate the long-run effects of these relative changes on pension risk-sharing.

The pension adjustment factors reflect current and projected generosity of the pension systems, and are also closely related to GDP per capita. In almost all countries, except Luxembourg (which has implemented no reforms to its pension system), the adjustment factors fall over time, reflecting the realization of reforms legislated by 2018 intended to curtail the generosity of pensions in the future.

Results for the mortality model are shown in Table 4.

Panel A shows the leading model, which is fitted to the pooled mortality of the Eurozone. The first two columns show the mean changes in the intercept (first column) and the slope (second column) of the mortality shape parameters by cohort. Note that the intercept falls, but that the projected changes in the slope are very small. This is consistent with the hypothesis of Vaupel (2010), and results for the US in McCarthy (2021), that most recent mortality improvements have been in the form of postponement of mortality, rather than rectangularization or compression. The next four columns contain the coefficients of the VAR model. An important

**Table 4**
Fitted BMAP mortality model parameters.

Panel A: Eurozone ('leading' model)

| Country | $\widehat{A}_1$ | $\widehat{A}_2$ | $\widehat{B}_{11}$ | $\widehat{B}_{12}$ | $\widehat{B}_{21}$ | $\widehat{B}_{22}$ | $\rho(\hat{B})$ | Prob (True model is stable)‡ |
|---|---|---|---|---|---|---|---|---|
| EUROZONE | −0.021 | 0.000 | −0.238 | 10.385 | 0.005 | 0.694 | 0.742 | 1.0000 |

Panel B: Country models ('following' models)

| Country | $\widehat{C}_1$ | $\widehat{C}_2$ | $\widehat{D}_{11}$ | $\widehat{D}_{12}$ | $\widehat{D}_{21}$ | $\widehat{D}_{22}$ | $\rho(\hat{D})$ | Prob (True model is stable)‡ |
|---|---|---|---|---|---|---|---|---|
| Austria | −0.002 | 0.000 | 0.964 | 1.725 | 0.005 | 0.891 | *1.029 | 0.5408 |
| Belgium | 0.023 | 0.000 | 0.273 | −11.265 | −0.008 | 0.726 | 0.877 | 0.9998 |
| Estonia | 0.026 | 0.002 | 0.881 | −1.640 | −0.015 | 0.803 | *1.005 | 0.7614 |
| Finland | −0.009 | −0.004 | 1.063 | −0.437 | 0.016 | 0.211 | *1.055 | 0.1154 |
| France | −0.002 | 0.000 | 0.966 | 1.376 | 0.006 | 0.975 | *1.062 | 0.1739 |
| Germany | 0.104 | 0.000 | −0.353 | −10.367 | −0.003 | 0.996 | *1.019 | 0.6841 |
| Greece | −0.004 | 0.001 | 0.199 | −0.513 | 0.032 | 0.927 | 0.904 | 0.9983 |
| Ireland | 0.021 | 0.000 | 0.840 | 0.146 | −0.009 | 0.740 | 0.825 | 0.9999 |
| Italy | −0.003 | 0.000 | 0.974 | 1.494 | −0.001 | 0.964 | 0.970 | 0.9529 |
| Latvia | 0.012 | 0.001 | 1.001 | −0.064 | −0.007 | 0.821 | *1.004 | 0.5662 |
| Lithuania | 0.011 | 0.000 | 0.972 | 1.665 | −0.003 | 0.871 | 0.923 | 0.9936 |
| Luxembourg | 0.003 | −0.005 | 0.912 | −0.052 | 0.020 | 0.188 | 0.910 | 0.9967 |
| Netherlands | 0.006 | 0.003 | 0.982 | −0.291 | −0.007 | 0.499 | 0.987 | 0.8740 |
| Portugal | 0.016 | 0.000 | 0.677 | 2.067 | 0.014 | 0.626 | 0.823 | 1.0000 |
| Slovakia | 0.169 | 0.003 | 0.511 | −12.583 | −0.011 | 0.729 | 0.999 | 0.7433 |
| Slovenia | 0.177 | −0.007 | 2.029 | 43.866 | −0.078 | −1.721 | 0.440 | 1.0000 |
| Spain | −0.020 | 0.001 | 0.705 | −3.136 | 0.005 | 0.888 | 0.801 | 0.9998 |

NOTE: †$\rho(\bullet)$ is the spherical radius of matrix •, the norm of the largest eigenvalue of •. The estimated VAR is stable iff $\rho(\bullet) < 1$. ‡$\rho(\bullet)$ is an estimated quantity and therefore subject to sampling error. This column reports the probability that the data was generated by an underlying model that is stable, using an approximate hypothesis test developed in Appendix A. Where the estimated VAR is unstable, the norm of the largest eigenvalue of • was set to 0.999 to ensure stability. Table shows unadjusted coefficients, marked with *.

quantity governing the stability of the VAR is the spherical radius of the matrix B, which is shown in the final column. The spherical radius equals the norm of the largest eigenvalue. If it is less than 1, then the VAR is stable.

The next panel shows the 'following' models for each country. These models are VAR's on the difference between the country-level model and the 'leading' model. Once again, the first column shows the change in the intercept coefficient and the second the change in the slope coefficient. Changes in the intercept are forecast to be much larger than changes in the slope. The next four columns show the coefficients of the VAR model for the differences between the 'following' models and the leading model. If the VAR of these differences is stable, then the mortality pattern of that country is coherent with the pooled population in the sense of Li and Lee (2005). The spherical radius of each coefficient matrix is once again shown in the second-to-last column. For all but 6 countries, the estimated spherical radius is less than 1, indicating that the model is stable. For a further 4 countries, the spherical radius is very slightly greater than 1. We note that the spherical radius is an estimated quantity, and therefore subject to sampling error. As outlined in appendix A, we use a simulation approach to estimate the probability that the true underlying model is stable, conditional on the observed values in panel B. These probabilities are shown in the final column. For all of the countries in the sample, we are unable to reject the hypothesis that the VAR's are stable at a confidence level of 5%. For these countries, for projection purposes, we therefore altered the matrices D slightly by performing an eigenvalue decomposition, setting the largest eigenvalue to have a norm of 0.999, and recalculating the matrices D. Table 4 shows unadjusted values of the elements of D.[14]

[14] While this approach may seem odd, the implication of non-stable VAR's for the following models is that mortality in those countries will continue to diverge from that of the Eurozone as a whole. Under a scenario of increasing political and economic integration, this seems highly unlikely.

Fig. 1 shows the fitted values of the parameters $\beta_c^{g,0} + b_c^{i,0}$ for the different countries, and for the Eurozone as a whole, along with cohort-by-cohort OLS estimates. The leading model is shown in the top left block. As can be seen from the figure, the BMAP estimates are much more stable than the cohort-by-cohort estimates. Fig. 2 shows the equivalent values of the slope, $\beta_c^{g,1} + b_c^{i,1}$. In most countries, changes in the slope are relatively small over the period, as in the case of the Eurozone as whole, although there are notable changes in the post-Warsaw pact countries.

Table 5 shows resulting mortality projections of remaining period life expectancy at age 55 for the different countries, along with Eurostat projections.

Fig. 3 shows the range of life expectancy at 2070, along with Eurozone projections. The whiskers of the box-and-whisker plots show the 95% confidence intervals, and the solid line in the center the medians. As can be seen, our results are reasonable in comparison to Eurostat projections, since these lie within our 95% CI's for most of the countries, although almost always toward the lower end of it, indicating the Eurostat is projecting slightly lower increases in longevity than we are. In particular, our projection suggest that Italy will have a life expectancy that increases much more rapidly than Eurostat is projecting. This is likely the consequence of rapid improvements in the mortality of recent cohorts (seen in Fig. 1) in Italy, relative to other countries, which the cohort-based method is projecting will endure for the remainder of the lives of these cohorts. Eurostat does not use a cohort-based approach when projecting mortality.

Table 6 and Fig. 4 show the projected pension expenditures of each country, as a percentage of GDP, along with Ageing Working Group projections. Once again, the AWG projections lie within our 95% CI, but near the bottom end for most countries, which is consistent with our longevity projections being more optimistic than Eurostat. Note that these models show the confidence intervals assuming no further parametric reforms to pension systems other than those enshrined in current law in 2018 and makes the as-

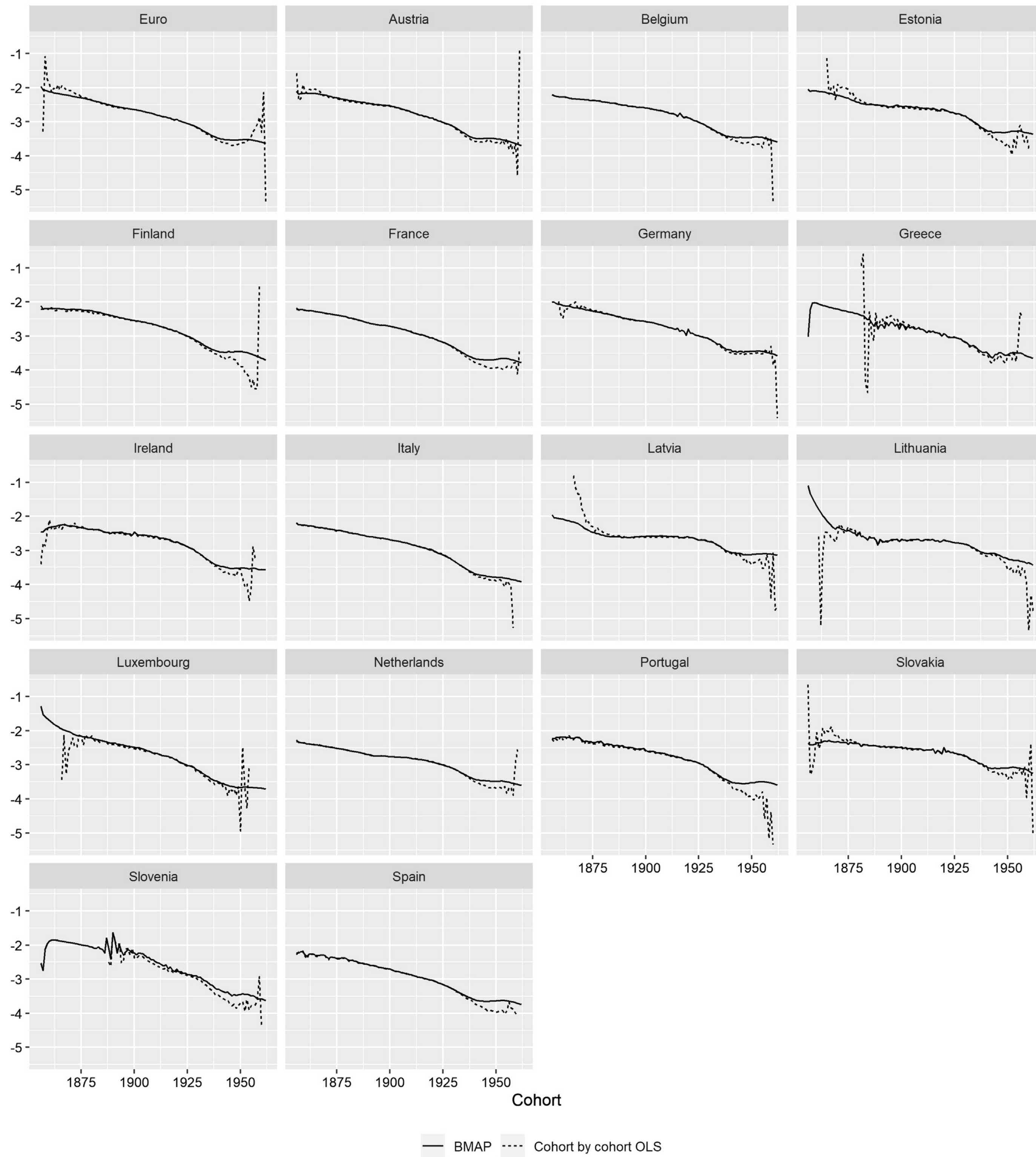



**Fig. 1.** The fitted values of the parameters $\beta_c^{g,0} + b_c^{i,0}$ for the different countries, and for the Eurozone as a whole, along with cohort-by-cohort OLS estimates.

sumption that these reforms are implemented as provided for in their enabling legislation. The range represents only the effect of mortality risk. All other sources of risk affecting pension expenditures as a percentage of GDP are ignored.[15]

Fig. 5 shows the pension betas calculated based on the projection of pension expenditure. Panel A shows the betas under the CAPM risk adjustment scheme, and Panel B shows those under the proportional benefit risk adjustment scheme. Betas are almost al-

[15] There are many other sources of risk: retirement behavior, unanticipated changes in GDP, changes in inflation and changes in productivity growth are all significant sources of risk that would increase the uncertainty of pension expenditures, but which are not shown in the figure.

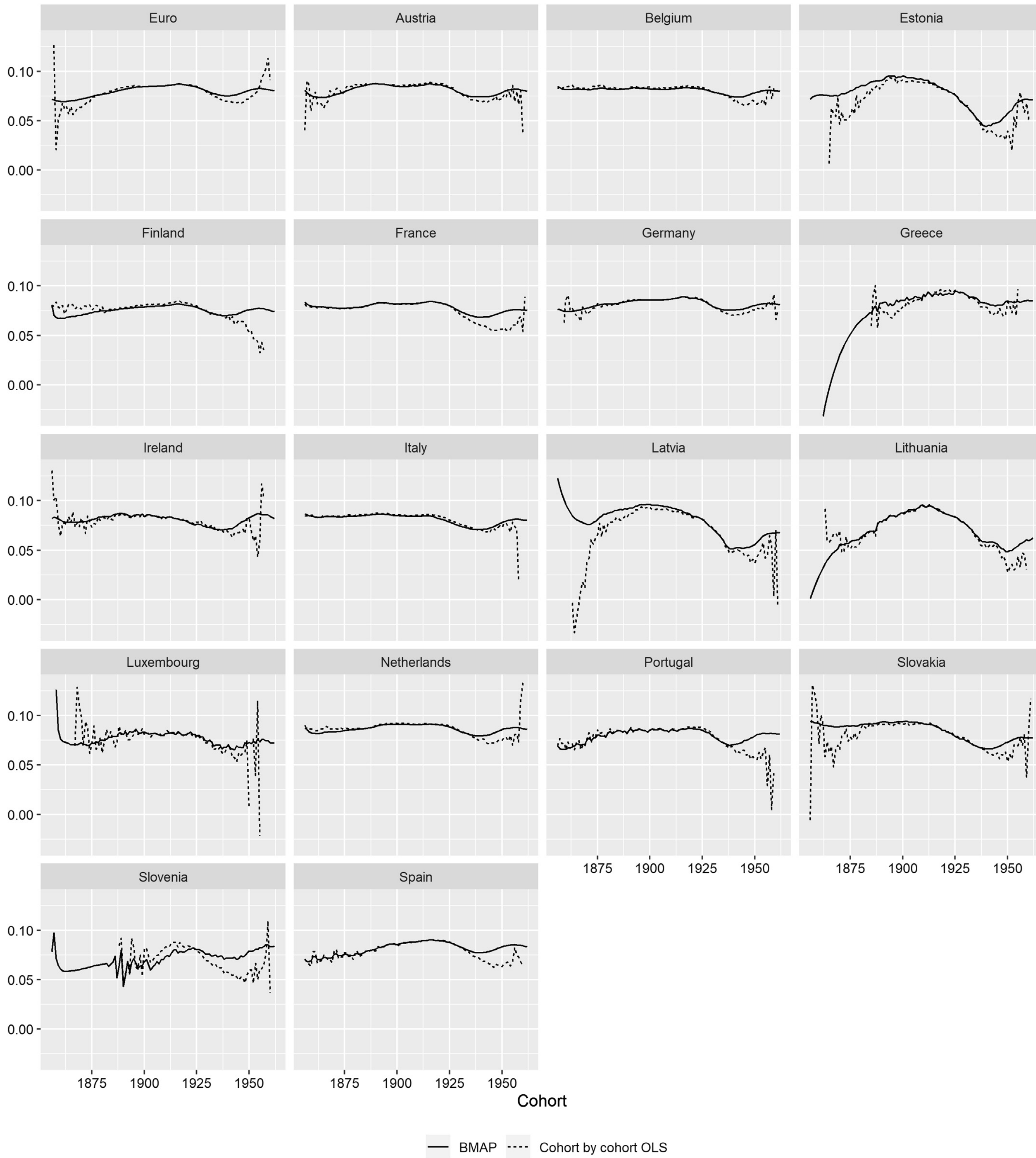



**Fig. 2.** The fitted values of the parameters $\beta_c^{g,1} + b_c^{i,1}$ for the different countries, and for the Eurozone as a whole, along with cohort-by-cohort OLS estimates.

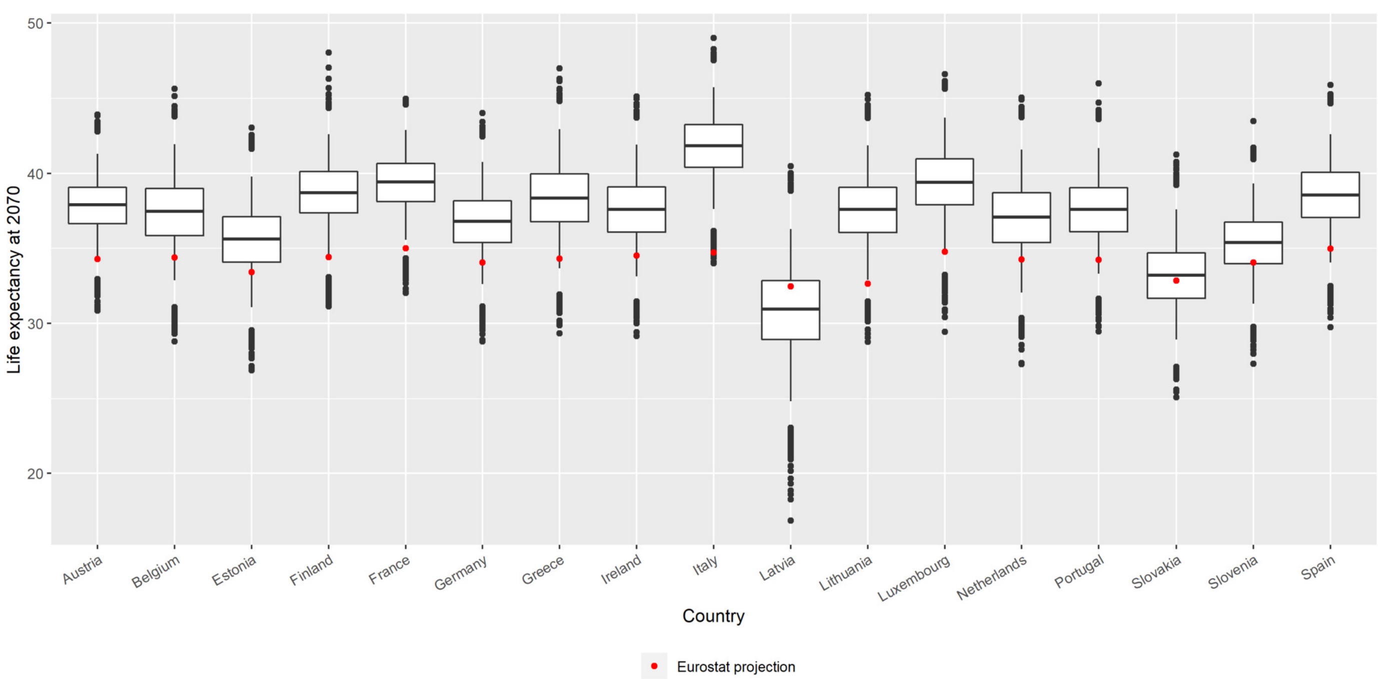
ways positive, and range from around 0 to around 2. These record the sensitivity of national pension expenditure to changes in the aggregate pension expenditure. A high CAPM beta implies that if aggregate Eurozone pension expenditure increases, the national expenditure is likely to increase by more, and vice versa. National pension expenditures may be more sensitive to Eurozone pension expenditures for two reasons: firstly because the national pension system is more generous than the EU average, and so a given mortality change has a larger effect on the country's pension expenditure than on the EU's aggregate pension expenditure, and secondly because a country's mortality may be more highly correlated with the EU average.

To separate these effects, we regress the logged CAPM betas on pension generosity, measured by the AWG projected pension expenditure as a % of GDP, in logs, with year and country fixed effects. (Negative values of beta were excluded.) The results are

**Table 5**
Cohort model projected remaining life expectancy (on a period basis) at 55 (with 95% confidence intervals), by country and for the Eurozone, 2030, 2050, and 2070. Eurostat projections shown for reference.

| Country | Remaining period life expectancy at age 55 (years) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2030 | | | | 2050 | | | | 2070 | | | |
| | 2.5% | 50% | 97.5% | Eurostat | 2.5% | 50% | 97.5% | Eurostat | 2.5% | 50% | 97.5% | Eurostat |
| Austria | 28.8 | 29.5 | 30.2 | 30.2 | 33.1 | 34.4 | 35.7 | 32.4 | 34.2 | 37.9 | 41.3 | 34.3 |
| Belgium | 28.6 | 29.5 | 30.3 | 30.3 | 31.6 | 33.4 | 35.0 | 32.5 | 32.9 | 37.4 | 41.9 | 34.4 |
| Estonia | 26.4 | 27.1 | 27.8 | 28.2 | 28.5 | 31.2 | 33.4 | 31.0 | 31.1 | 35.6 | 39.8 | 33.4 |
| Finland | 28.4 | 29.8 | 31.2 | 30.4 | 32.8 | 34.8 | 36.7 | 32.5 | 34.6 | 38.7 | 42.6 | 34.4 |
| France | 29.7 | 30.7 | 31.7 | 31.4 | 33.9 | 35.4 | 36.9 | 33.3 | 35.6 | 39.4 | 42.9 | 35.0 |
| Germany | 27.6 | 28.2 | 28.9 | 29.7 | 31.2 | 32.9 | 34.4 | 32.0 | 32.6 | 36.8 | 40.8 | 34.0 |
| Greece | 29.6 | 30.1 | 30.5 | 30.1 | 32.8 | 34.4 | 35.8 | 32.3 | 33.7 | 38.4 | 42.9 | 34.3 |
| Ireland | 29.4 | 30.2 | 31.0 | 30.6 | 32.0 | 33.4 | 34.8 | 32.7 | 33.1 | 37.6 | 41.9 | 34.5 |
| Italy | 31.2 | 32.3 | 33.5 | 31.1 | 35.9 | 37.6 | 39.3 | 33.0 | 37.6 | 41.8 | 45.7 | 34.7 |
| Latvia | 24.0 | 24.8 | 25.7 | 26.3 | 23.6 | 27.4 | 30.5 | 29.6 | 24.8 | 31.0 | 36.3 | 32.5 |
| Lithuania | 24.7 | 25.5 | 26.4 | 26.8 | 30.9 | 33.4 | 35.7 | 29.9 | 32.9 | 37.6 | 41.9 | 32.6 |
| Luxembourg | 29.7 | 30.7 | 31.7 | 31.0 | 33.5 | 35.3 | 37.2 | 33.0 | 34.8 | 39.4 | 43.7 | 34.8 |
| Netherlands | 28.3 | 29.4 | 30.5 | 30.2 | 31.2 | 33.2 | 35.0 | 32.3 | 32.1 | 37.1 | 41.6 | 34.2 |
| Portugal | 29.1 | 30.0 | 30.9 | 30.3 | 32.3 | 33.7 | 35.0 | 32.4 | 33.3 | 37.6 | 41.7 | 34.2 |
| Slovakia | 24.9 | 25.6 | 26.2 | 27.2 | 27.6 | 29.5 | 31.2 | 30.2 | 28.9 | 33.2 | 37.6 | 32.8 |
| Slovenia | 28.2 | 28.8 | 29.3 | 29.8 | 29.9 | 31.6 | 33.1 | 32.0 | 31.3 | 35.4 | 39.3 | 34.0 |
| Spain | 29.4 | 30.8 | 32.2 | 31.5 | 32.6 | 34.7 | 36.6 | 33.3 | 34.0 | 38.6 | 42.6 | 35.0 |

Note: The table shows the projected remaining life expectancy at age 55 in each country (on a period basis) produced by the coherent cohort-based model. Confidence intervals based on a simulation of 10,000 runs for each country and for the Eurozone. Details of the model and the estimation procedure in the text.




Note: The box-whiskers plot shows the following statistics of the cohort-based model projection for countries: 97.5th percentile (top hinge), 75th percentile (box top), median (middle bar), 25th percentile (box bottom), and 2.5th percentile (bottom hinge). Red dots show the Eurostat projection, which mostly lie in the bottom quartile of our projections.

**Fig. 3.** Projected remaining period life expectancy at age 55 for the 17 countries at 2070. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

shown in Table 7. The elasticity of pension generosity to pension risk (CAPM beta) is 3.149, meaning that for a 1% increase in pension generosity, beta will increase 3.149% on average. With omitted country France, the country fixed effects show the relative (to France) sensitivity to mortality risk, given the same pension generosity. Fig. 6 shows a graphical representation of these fixed effects. We see Belgium, Greece, Ireland, Latvia, Lithuania, Slovakia, and Spain are not significantly different from France in sensitivity to mortality risk. But Austria, Estonia, Finland, Italy, Luxembourg, Portugal, and Slovenia are significantly lower, and Germany, and the Netherlands are significantly higher.

Tables 8, 9 and 10 show the Eurozone-wide and country-specific benefits of pooling the mortality risk in SS liabilities, using the risk measures discussed in the previous section. Under all premium structures there is a reduction in the aggregate standard deviation of pension expenditure of 0.11% p.a. of GDP across the Eurozone at the maximum, in around 2050. Note that a reduction of 0.11% of the standard deviation is equivalent to around 0.33% p.a. of GDP reduction in the 99th percentile (making the assumption that these expenditures are approximately normally distributed). This represents around 3% of average pension expenditure across the Eurozone, and 1% of average tax revenue. The

**Table 6**
Projected pension expenditures (as a % of country/Eurozone GDP) by country and for the Eurozone, 2030, 2050, and 2070. AWG projections shown for reference.

| Country | Projected pension expenditures (% of current country/Eurozone GDP) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2030 | | | | 2050 | | | | 2070 | | | |
| | 2.5% | 50% | 97.5% | AWG | 2.5% | 50% | 97.5% | AWG | 2.5% | 50% | 97.5% | AWG |
| Austria | 14.7 | 14.9 | 15.0 | 14.1 | 15.0 | 15.4 | 15.9 | 14.2 | 14.7 | 16.1 | 17.4 | 13.9 |
| Belgium | 14.7 | 15.0 | 15.2 | 13.8 | 15.3 | 16.2 | 16.9 | 14.7 | 14.9 | 18.3 | 21.7 | 15.0 |
| Estonia | 7.4 | 7.5 | 7.6 | 7.2 | 6.8 | 7.2 | 7.6 | 7.1 | 5.9 | 6.7 | 7.5 | 6.4 |
| Finland | 15.2 | 15.7 | 16.2 | 14.8 | 14.2 | 15.0 | 15.7 | 13.2 | 14.9 | 16.8 | 18.5 | 13.9 |
| France | 16.0 | 16.3 | 16.6 | 15.4 | 14.7 | 15.1 | 15.5 | 13.8 | 12.5 | 13.9 | 15.1 | 11.8 |
| Germany | 11.9 | 12.1 | 12.3 | 11.5 | 12.2 | 12.9 | 13.4 | 12.2 | 12.2 | 14.1 | 15.7 | 12.5 |
| Greece | 12.7 | 12.9 | 13.0 | 12.0 | 12.9 | 13.5 | 14.0 | 12.5 | 10.6 | 12.9 | 15.0 | 10.6 |
| Ireland | 4.3 | 4.3 | 4.3 | 4.3 | 5.6 | 5.8 | 6.0 | 6.1 | 4.9 | 5.7 | 6.3 | 6.0 |
| Italy | 19.2 | 19.5 | 19.8 | 17.2 | 20.1 | 20.8 | 21.4 | 17.3 | 17.1 | 19.6 | 21.7 | 13.9 |
| Latvia | 6.3 | 6.4 | 6.5 | 6.2 | 5.2 | 5.8 | 6.3 | 6.1 | 3.5 | 4.4 | 5.3 | 4.7 |
| Lithuania | 7.3 | 7.5 | 7.6 | 7.1 | 6.9 | 7.4 | 7.9 | 6.5 | 5.6 | 6.4 | 7.1 | 5.2 |
| Luxembourg | 10.8 | 10.9 | 11.1 | 10.2 | 13.6 | 14.2 | 14.7 | 13.0 | 18.6 | 20.9 | 23.0 | 17.9 |
| Netherlands | 7.7 | 7.8 | 8.0 | 7.5 | 8.3 | 8.7 | 9.1 | 8.2 | 7.5 | 9.2 | 10.6 | 7.9 |
| Portugal | 15.5 | 15.8 | 16.0 | 14.3 | 14.5 | 15.0 | 15.5 | 13.7 | 11.7 | 14.2 | 16.5 | 11.4 |
| Slovakia | 7.6 | 7.6 | 7.7 | 7.6 | 8.2 | 8.7 | 9.1 | 8.8 | 8.4 | 9.8 | 11.1 | 9.8 |
| Slovenia | 12.4 | 12.5 | 12.6 | 12.0 | 15.1 | 15.9 | 16.6 | 15.6 | 13.7 | 15.8 | 17.8 | 14.9 |
| Spain | 12.8 | 13.0 | 13.3 | 12.6 | 13.7 | 14.4 | 14.9 | 13.9 | 10.6 | 11.9 | 13.0 | 10.7 |
| EUROZONE | 13.8 | 13.9 | 14.0 | 13.0 | 14.0 | 14.3 | 14.6 | 13.1 | 12.2 | 13.9 | 15.3 | 11.7 |

Note: The table shows the projected pension expenditures as a % of country/Eurozone GDP. Confidence intervals based on a simulation of 10,000 runs of mortality risk. Details of the model and the estimation procedure in the text.

**Table 7**
Regression results of regressing log(CAPM betas) on log(pension generosity) with country and year fixed effects.

| | Estimate | Std dev | t-stats | p-value | |
|---|---|---|---|---|---|
| log(Pension generosity) | 3.149 | 0.276 | 11.411 | 0.000 | *** |
| Austria | −1.687 | 0.167 | −10.071 | 0.000 | *** |
| Belgium | −0.185 | 0.158 | −1.169 | 0.243 | |
| Estonia | −0.661 | 0.260 | −2.543 | 0.011 | ** |
| Finland | −1.089 | 0.166 | −6.544 | 0.000 | *** |
| Germany | 0.579 | 0.161 | 3.591 | 0.000 | *** |
| Greece | −0.212 | 0.162 | −1.311 | 0.190 | |
| Ireland | 0.239 | 0.302 | 0.791 | 0.429 | |
| Italy | −0.311 | 0.164 | −1.897 | 0.058 | * |
| Latvia | −0.180 | 0.325 | −0.552 | 0.581 | |
| Lithuania | 0.194 | 0.278 | 0.697 | 0.486 | |
| Luxembourg | −1.493 | 0.158 | −9.444 | 0.000 | *** |
| Netherlands | 0.990 | 0.218 | 4.541 | 0.000 | *** |
| Portugal | −0.907 | 0.158 | −5.754 | 0.000 | *** |
| Slovakia | 0.173 | 0.200 | 0.865 | 0.387 | |
| Slovenia | −1.199 | 0.163 | −7.355 | 0.000 | *** |
| Spain | 0.215 | 0.160 | 1.343 | 0.180 | |
| Year fixed effects | X | | | | |
| Adj R-squared: 0.69 | | | | | |
| Observations: 930 | | | | | |

Note: The table shows the regression results of regressing country betas on pension generosity, in logs, with country and year fixed effects. The elasticity of pension generosity to pension risk appears to be 3.032. The country fixed effects coefficients show the relative (to France) sensitivity to mortality risk as if all countries have the same pension generosity. Negative values of beta were excluded from the regression.

pattern of aggregate benefits of risk pooling, shown in the first line of Tables 8, 9 and 10 is striking: pooling benefits rise over time as a percentage of GDP, until around 2050, and then fall again.

The insight underlying this phenomenon is two-fold. First, benefits of risk pooling rise as the amount of uncertainty rises. Because mortality rates are reasonably predictable at short horizons, this means that the benefits of risk-pooling at short horizons are small. As the time horizon lengthens, the amount of uncertainty rises, and the potential benefits rise too. The second issue is the correlation between the mortality risk of different countries. As the correlation rises, the benefits to risk pooling falls. But in our model, because all countries follow the leading model, correlation between countries rises inexorably over time (provided the long-term trend is different from zero, which we find it is). The precise trade-off between these two factors is therefore an empirical question.

The importance of these two factors is well illustrated by Figs. 7 and 8, which show scatterplots of pension expenditures as a fraction of GDP in 2030 and 2070 for a selection of countries. Two things are notable: first, the difference in the scale between the two sets of graphs, and second the difference in correlations. In 2030, the vertical and horizontal scales are limited to a few fractions of a percent of GDP. In 2070, the scales can be as large as 5% of GDP. This illustrates the first point, regarding the degree of aggregate uncertainty, which rises over time. However, the second difference between the two graphs is more evident: the increase in correlation between the pension expenditures of different countries over time, driven by the increase in correlation in mortality rates. In 2030, the correlation is very low, so the scatter plots are spherical, but by 2070, cross-country correlations are as high as 97.6% (between Netherlands and Germany for instance), and the scatter plots look in some cases like straight lines. The reason for this is the assumption of mortality coherence: because all countries share the same mortality trend, as time goes on, the trend (the 'leading' model) begins to dominate over the uncertainty between countries (the 'following' models).

In aggregate, our results show that, at first, the increasing uncertainty factor wins out: the benefits to pooling increase. After about 30 years out, however, the increasing correlation begins to dominate, and the benefits of pooling decrease again. Note that because we ignore correlation between countries in the period effects, the benefits of risk pooling are likely overstated, especially in the very short term. Over time, however, the significance of these period effects likely falls as other sources of variation begin to dominate.

Figs. 9 and 10 show a graphical representation of these pooling benefits. S1 indicates the naïve case where aggregate risk is allocated according to GDP, S2 indicates the CAPM risk-allocation, and S3 represents the proportional-benefit risk allocation. Fig. 9 shows that the aggregate pooling benefits are similar in all cases, with the increasing and then decreasing benefit as discussed. But Fig. 10 shows that the net benefits of risk pooling are distributed across countries very differently under the three premium systems. Under naïve premiums (top panel, using S1), some countries gain tremendously from risk pooling (notably Belgium, Portugal and Slovenia), while others lose (Ireland, Lithuania and Estonia). Those countries

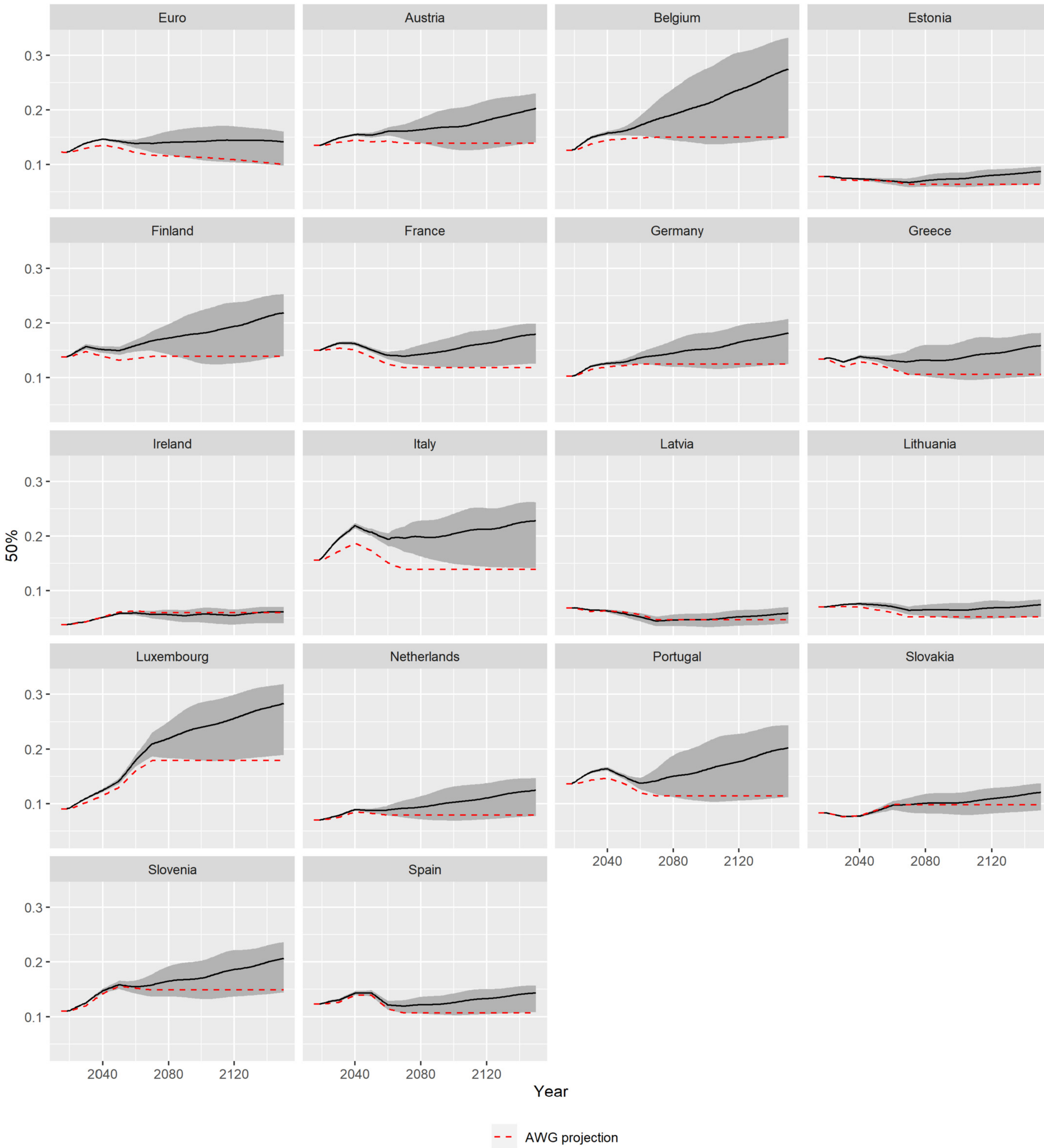


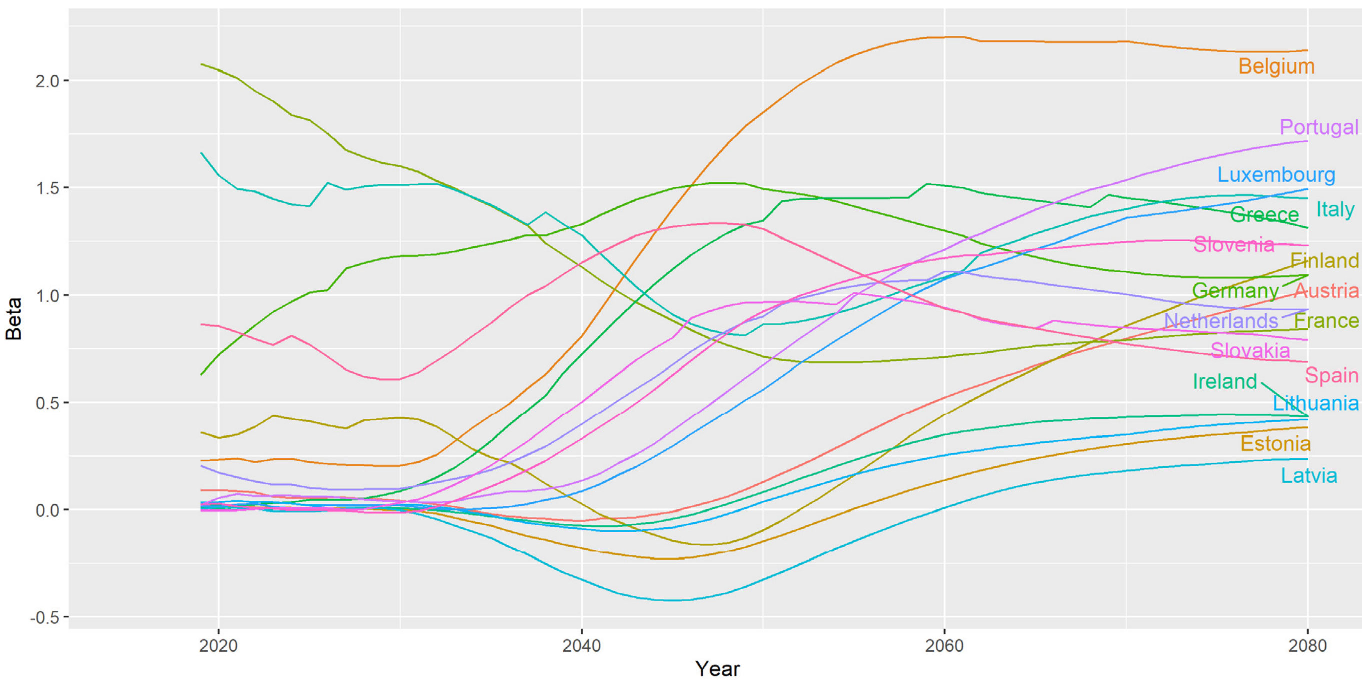

**Fig. 4.** 95% confidence intervals of pension expenditure/GDP of countries and the Eurozone as a whole.

with more generous pension systems as a proportion of GDP tend to gain from naïve risk pooling, while those with less generous pension systems tend to lose. Under CAPM risk-allocation (middle panel, using S2), however, all countries gain from risk-pooling, with smaller countries tending to gain the most. Under S3 (bottom panel), by design, all countries benefit proportionally from participation in the risk pool, but smaller countries and larger countries benefit by an equal proportional reduction in standard deviations as a percentage of GDP. These results are discussed further in the next section.

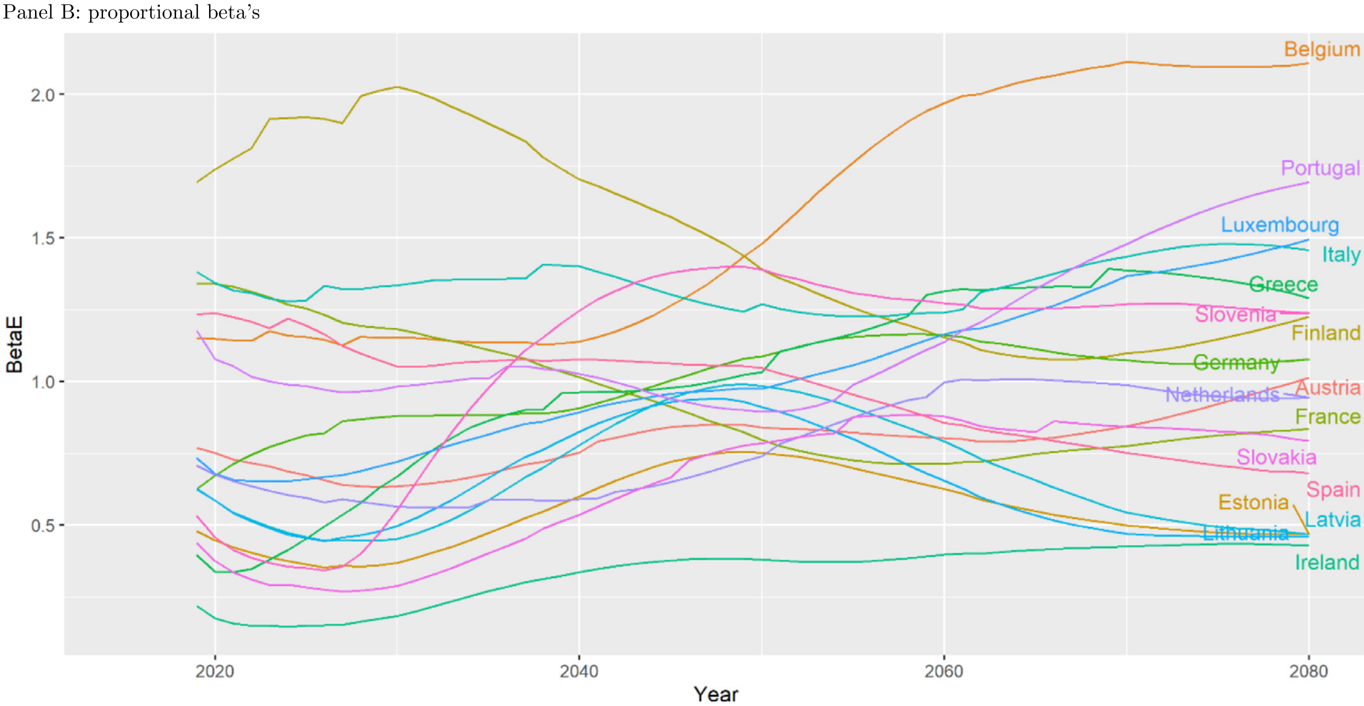
## 5. Discussion and conclusion

In the previous section, we showed that the pattern over time of the aggregate benefits to risk-pooling is the consequence of two competing factors. First, the amount of aggregate mortality uncertainty rises over time, increasing the potential benefits of pooling.

Panel A: CAPM beta's

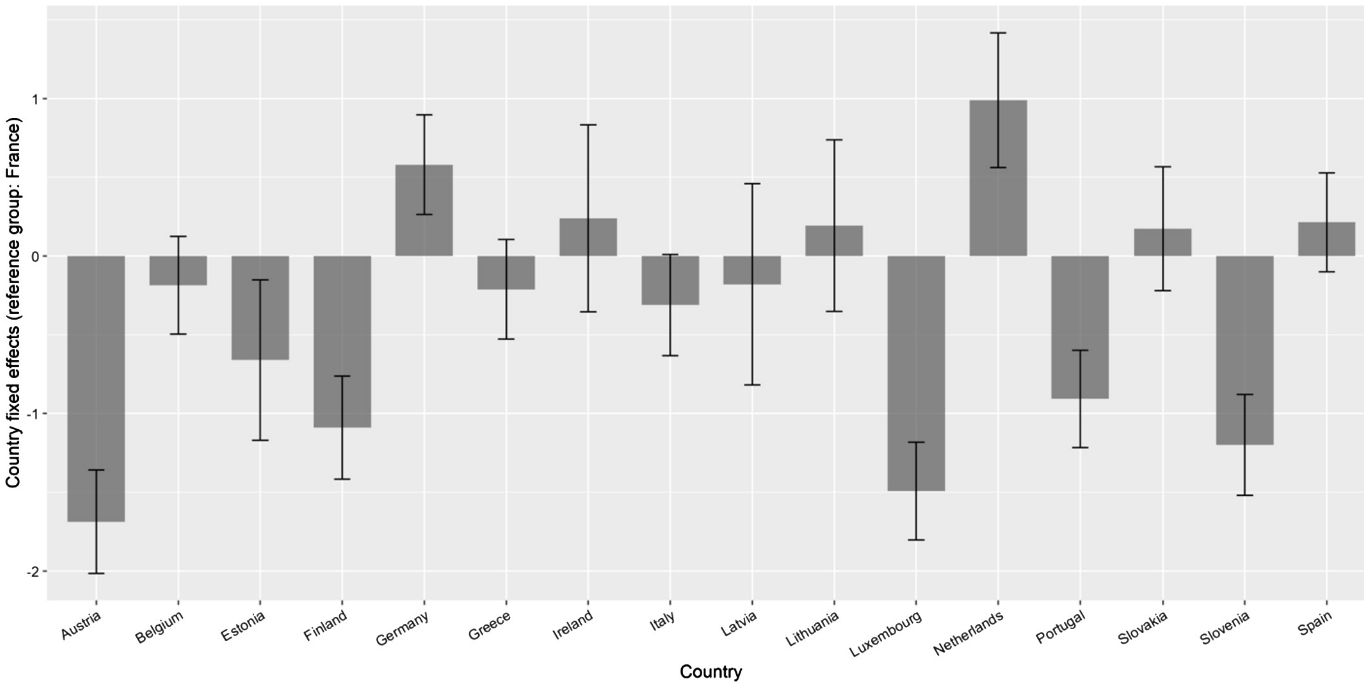



Panel B: proportional beta's




NOTE: The figure shows the pension system betas estimated using projected pension expenditure. Panel A shows the CAPM betas where each country pays premiums equal to their contribution of risk to the pool, while panel B shows betas derived so as to ensure that each country enjoys the same proportional reduction in the standard deviation of their pension expenditures as a percentage of GDP as a result of participating in the pool. In both models, low betas are the consequence of either the generosity of the pension system, or the low sensitivity of the country's mortality to the aggregate Eurozone mortality.

**Fig. 5.** Pension system betas estimated using projected pension expenditure. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)




NOTE: The figure shows the country fixed effects coefficients and confidence intervals in the regression of sensitivity to mortality risk (reference group: France). We regress country betas on pension generosity, in logs, with country and year fixed effects. This shows the relative (to France) sensitivity to mortality risk as if all countries have the same pension generosity.

**Fig. 6.** Country fixed effects coefficients in the regression of logged pension system betas on logged pension generosity (reference group: France).

**Table 8**
The benefits of pooling SS mortality risk using unadjusted premiums (benefits measured using the reduction in standard deviation of pension expenditures), % GDP.

| | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | Notes |
|---|---|---|---|---|---|---|---|
| Full pooling | 0.02 | 0.07 | 0.09 | 0.11 | 0.08 | 0.05 | Figures show the sum of the country-level pooling benefit, as a difference between the standard deviations of the pension expenditure before and after risk-pooling using risk-unadjusted premiums, as a % of the Eurozone aggregate GDP. |
| Austria | 0.01 | 0.02 | 0.05 | 0.06 | −0.01 | −0.08 | Figures show the difference between the country-level standard deviation of pension expenditure before and after risk-pooling using risk-unadjusted premiums, as a % of GDP. A positive number therefore indicates the risk-pooling reduces the standard deviation of pension expenditure. |
| Belgium | 0.03 | 0.08 | 0.12 | 0.24 | 0.55 | 0.97 | |
| Estonia | 0.00 | −0.01 | 0.02 | 0.04 | −0.10 | −0.36 | |
| Finland | 0.05 | 0.19 | 0.22 | 0.21 | 0.16 | 0.13 | |
| France | 0.03 | 0.09 | 0.10 | 0.05 | −0.06 | −0.13 | |
| Germany | 0.01 | 0.05 | 0.08 | 0.13 | 0.16 | 0.11 | |
| Greece | 0.00 | 0.03 | 0.09 | 0.11 | 0.23 | 0.37 | |
| Ireland | −0.01 | −0.03 | −0.02 | −0.06 | −0.21 | −0.42 | |
| Italy | 0.03 | 0.11 | 0.16 | 0.18 | 0.20 | 0.41 | |
| Latvia | 0.01 | 0.00 | 0.06 | 0.10 | −0.02 | −0.32 | |
| Lithuania | 0.01 | 0.01 | 0.06 | 0.08 | −0.08 | −0.39 | |
| Luxembourg | 0.01 | 0.03 | 0.08 | 0.10 | 0.16 | 0.35 | |
| Netherlands | 0.01 | 0.02 | 0.02 | 0.03 | 0.08 | 0.04 | |
| Portugal | 0.02 | 0.06 | 0.10 | 0.08 | 0.15 | 0.44 | |
| Slovakia | 0.00 | −0.02 | 0.01 | 0.05 | 0.02 | −0.08 | |
| Slovenia | 0.00 | 0.01 | 0.14 | 0.21 | 0.21 | 0.27 | |
| Spain | 0.03 | 0.07 | 0.11 | 0.12 | 0.01 | −0.15 | |

Second, the increasing correlation between the mortality rates of different countries, driven by the fact that, over time, uncertainty in mortality trends dominate the country-specific deviations from the trend, reduces the benefits of diversification. Taken together, the first factor dominates for the first 30 years, leading to an increase in the benefits of risk-pooling until around 2050. After this point, however, the second factor dominates and the aggregate benefits begin to fall. This would suggest that the optimal contract should be limited to around 30 years' duration. Fortunately, this is around the maximum lifespan of individuals at the age of 70, an issue to which we return below.

However, our model also indicates that mortality risk and sensitivity to mortality risk are not the same across countries. Not all countries are net beneficiaries under simple risk pooling, especially at longer time horizons. Therefore, allocating the aggregate risk across countries in some way that reflects each country's underlying risk is necessary. Risk-adjusting premiums using CAPM-type betas, calculated by considering pension expenditure as a percentage of GDP in each country and across the Eurozone seems to work fairly well in ensuring that all countries are beneficiaries of risk-pooling, although benefits of risk-pooling are still not equally shared across countries. An alternative approach – allocating ag-

**Table 9**
The benefits of pooling SS mortality risk using CAPM risk-adjusted premiums (benefits measured using the reduction in standard deviation of pension expenditures), % GDP.

| | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | Notes |
|---|---|---|---|---|---|---|---|
| Full pooling | 0.02 | 0.07 | 0.09 | 0.10 | 0.08 | 0.05 | Figures show the sum of the country-level pooling benefit, as a difference between the standard deviations of the pension expenditure before and after risk-pooling using CAPM risk-adjusted premiums, as a % of the Eurozone aggregate GDP. |
| Austria | 0.03 | 0.07 | 0.13 | 0.21 | 0.18 | 0.08 | Figures show the difference between the country-level standard deviation of pension expenditure before and after risk-pooling using CAPM risk-adjusted premiums, as a % of GDP. A positive number therefore indicates the risk-pooling reduces the standard deviation of pension expenditure. |
| Belgium | 0.04 | 0.13 | 0.13 | 0.09 | 0.07 | 0.06 | |
| Estonia | 0.02 | 0.04 | 0.09 | 0.18 | 0.25 | 0.18 | |
| Finland | 0.06 | 0.22 | 0.30 | 0.37 | 0.38 | 0.24 | |
| France | 0.02 | 0.06 | 0.09 | 0.10 | 0.06 | 0.03 | |
| Germany | 0.01 | 0.04 | 0.05 | 0.05 | 0.04 | 0.03 | |
| Greece | 0.01 | 0.07 | 0.11 | 0.06 | 0.03 | 0.02 | |
| Ireland | 0.01 | 0.02 | 0.05 | 0.09 | 0.05 | 0.02 | |
| Italy | 0.02 | 0.08 | 0.14 | 0.20 | 0.16 | 0.10 | |
| Latvia | 0.02 | 0.05 | 0.11 | 0.22 | 0.38 | 0.31 | |
| Lithuania | 0.02 | 0.06 | 0.14 | 0.24 | 0.21 | 0.12 | |
| Luxembourg | 0.02 | 0.08 | 0.15 | 0.17 | 0.13 | 0.08 | |
| Netherlands | 0.02 | 0.06 | 0.07 | 0.05 | 0.04 | 0.04 | |
| Portugal | 0.04 | 0.11 | 0.17 | 0.13 | 0.06 | 0.03 | |
| Slovakia | 0.01 | 0.03 | 0.05 | 0.05 | 0.05 | 0.04 | |
| Slovenia | 0.02 | 0.06 | 0.19 | 0.23 | 0.14 | 0.08 | |
| Spain | 0.03 | 0.09 | 0.10 | 0.07 | 0.04 | 0.02 | |

**Table 10**
The benefits of pooling SS mortality risk using proportional allocation of aggregate diversification benefits risk-adjusted premiums (benefits measured using the reduction in standard deviation of pension expenditures), % GDP.

| | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | Notes |
|---|---|---|---|---|---|---|---|
| Full pooling | 0.02 | 0.07 | 0.09 | 0.11 | 0.08 | 0.05 | Figures show the sum of the country-level pooling benefit, as a difference between the standard deviations of the pension expenditure before and after risk-pooling using risk-adjusted premiums designed to spread the benefits of participation proportionally across countries, as a % of the Eurozone aggregate GDP. |
| Austria | 0.02 | 0.04 | 0.07 | 0.09 | 0.07 | 0.04 | Figures show the difference between the country-level standard deviation of pension expenditure before and after risk-pooling using risk-adjusted premiums risk-adjusted premiums designed to spread the benefits of participation proportionally across countries, as a % of GDP. A positive number therefore indicates the risk-pooling reduces the standard deviation of pension expenditure. |
| Belgium | 0.02 | 0.08 | 0.11 | 0.16 | 0.16 | 0.11 | |
| Estonia | 0.01 | 0.02 | 0.06 | 0.08 | 0.05 | 0.03 | |
| Finland | 0.04 | 0.13 | 0.16 | 0.15 | 0.09 | 0.06 | |
| France | 0.03 | 0.08 | 0.10 | 0.08 | 0.06 | 0.04 | |
| Germany | 0.01 | 0.06 | 0.09 | 0.12 | 0.10 | 0.05 | |
| Greece | 0.01 | 0.04 | 0.09 | 0.11 | 0.11 | 0.07 | |
| Ireland | 0.00 | 0.01 | 0.03 | 0.04 | 0.03 | 0.02 | |
| Italy | 0.03 | 0.09 | 0.13 | 0.13 | 0.10 | 0.07 | |
| Latvia | 0.01 | 0.03 | 0.07 | 0.10 | 0.07 | 0.03 | |
| Lithuania | 0.01 | 0.03 | 0.08 | 0.10 | 0.05 | 0.02 | |
| Luxembourg | 0.01 | 0.05 | 0.08 | 0.10 | 0.10 | 0.07 | |
| Netherlands | 0.01 | 0.04 | 0.06 | 0.08 | 0.08 | 0.05 | |
| Portugal | 0.02 | 0.07 | 0.10 | 0.09 | 0.09 | 0.08 | |
| Slovakia | 0.01 | 0.02 | 0.05 | 0.08 | 0.07 | 0.04 | |
| Slovenia | 0.01 | 0.04 | 0.12 | 0.15 | 0.10 | 0.06 | |
| Spain | 0.03 | 0.07 | 0.10 | 0.11 | 0.07 | 0.04 | |

gregate risk in a way that ensures a proportional distribution of diversification benefits – appears to be effective but may result in cross-subsidies between countries. Measuring country-specific benefits of risk pooling by discounting the premiums less claims at country level requires an appropriate risk-adjusted discount rate, a thorny issue discussed in footnote 8, that we leave for future research.

An important concern with sharing mortality risks in the risk-pooling arrangement we have proposed is moral hazard. Countries might, rightly, be concerned that pooling the risk in social security systems could lead to systematic transfers of pension expenditure between countries, and create a common-action problem by reducing the impetus of individual countries for pension reform. We now explore these various sources of moral hazard in more detail.

One important moral hazard springs from the fact that the risk-pooling mechanism we have outlined is model-dependent, due to the manner in which the pension system betas are estimated. Countries could therefore systematically pass pension costs onto other countries by altering their projection models to understate future expenditure. Institutional factors – such as using centrally-prepared Eurozone projections, cross-validating and publishing models – may, however, partially deal with this issue.[16] Eurostat already produces centralized population projections, and could fairly easily produce commonly-accepted projections of pension expenditure.

Another source of moral hazard is that some countries have reformed their pension systems, while others have not. Longevity insurance may reduce the incentives faced by countries to reform

[16] Another difficulty would be adjusting projections by socio-economic status. To the extent that mortality differences are widening across social-economic status, this could significantly influence payments. Ideally, mortality models would incorporate this feature into their projections.

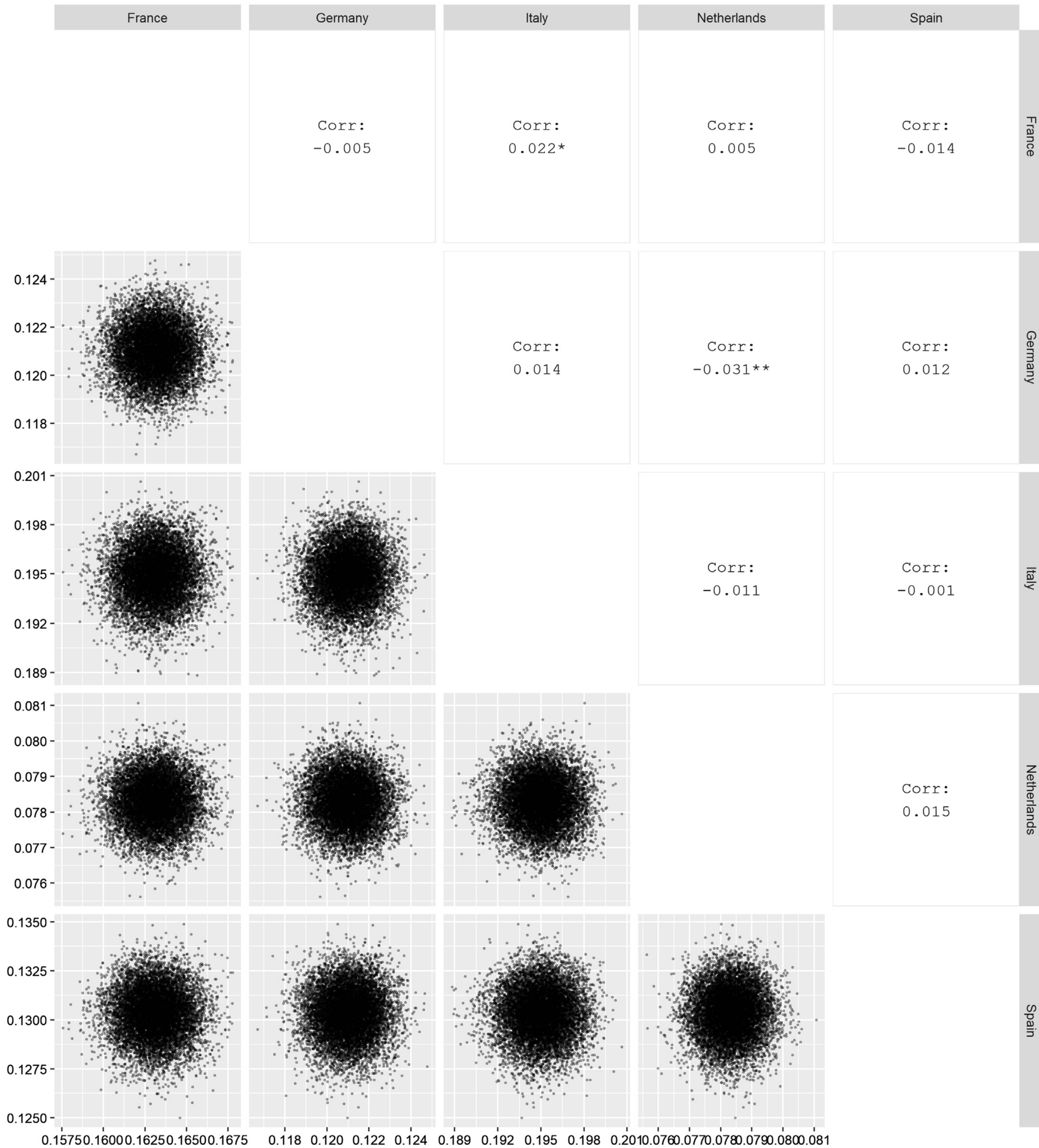



**Fig. 7.** Pairwise scatter plots of pension expenditure as a % of GDP in 2030, selected countries.

their pension systems, because in this case the costs of not reforming these systems are borne collectively, while the benefits are enjoyed individually. Any pooling of risk therefore needs to ensure that countries have the correct incentives to continue with parametric reforms as and when necessary. As discussed in Winter (2013), optimal contracting in the presence of moral hazard depends on the type of moral hazard involved. If there is moral hazard regarding the probability of a claim, the optimal contract is full insurance above a deductible. If, as is more likely to be the case here, there is moral hazard regarding the amount of a claim, then the optimal contract is partial insurance, or full insurance up to a cap. In this case, the latter is probably better as it would leave the costs and benefits of changes in pension amounts at the margin with the country itself, whereas partial insurance would reduce the incentives for reform away from first-best.

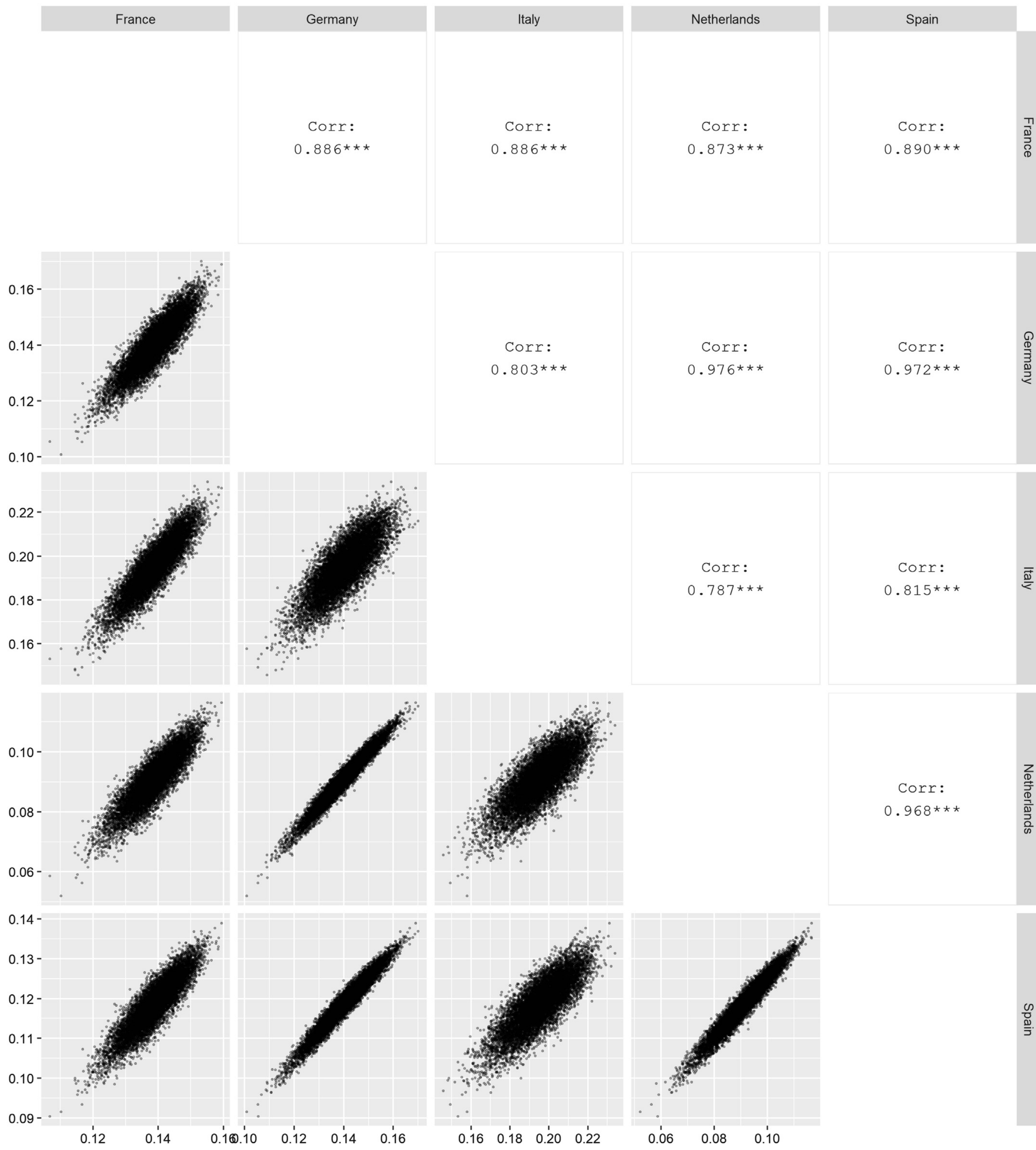
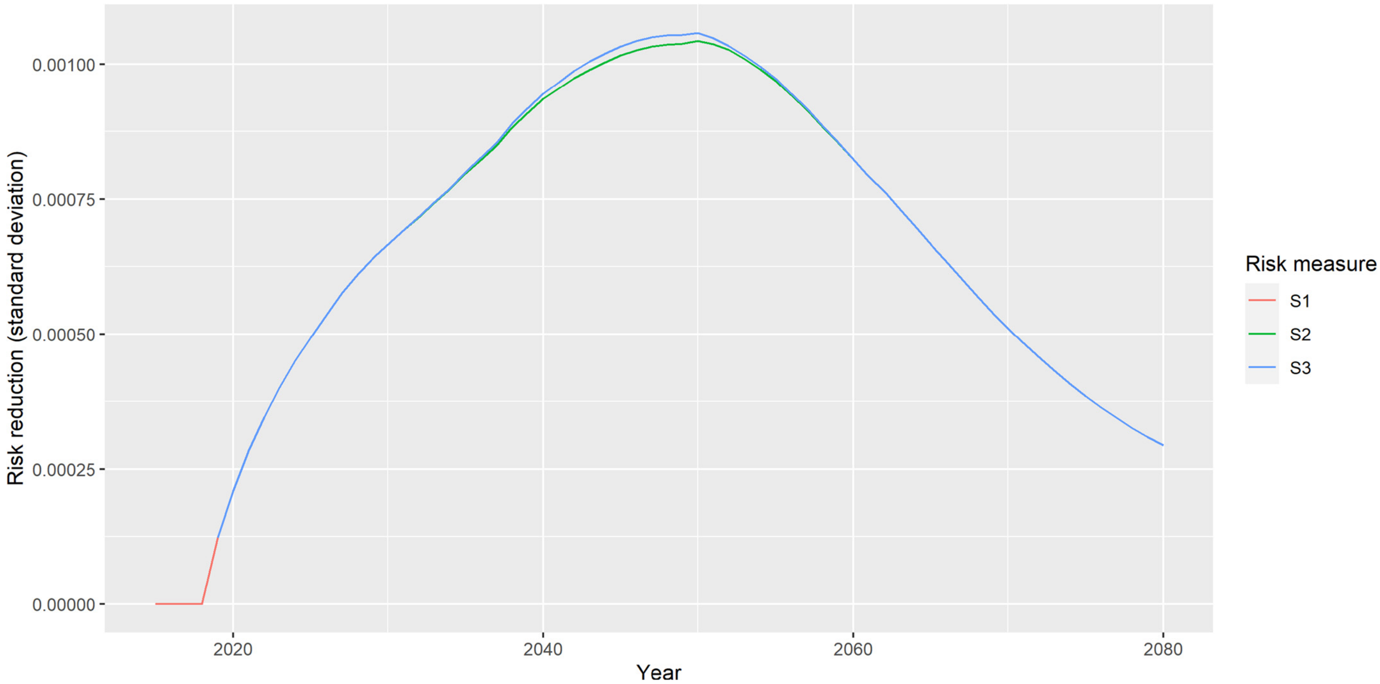



**Fig. 8.** Pairwise scatter plots of pension expenditure as a % of GDP in 2070, selected countries.

In most countries, state pensions are not entitlements, and are not viewed as contractual obligations of the state. Rather, they are viewed as expenditure, and therefore subject to change, as, for example in Greece, where pensions in payment were reduced substantially to assist in the stabilization of the national finances following that country's fiscal crisis. This creates challenges for risk pooling as the pool would need to be robust to contractual changes. One possibility is that records could be kept of the individuals and the pension amounts that are included in the pool, similar to securitizations that record the exact mortgages that are included in the structure. Then, regardless of parametric changes, payments can be based on the pensions that would have been paid to those exact individuals on the assumptions that were initially made. In this way, national governments would be made responsible for any differences at the margin caused by parametric reforms, again preserving first-best incentives for reform.




NOTE: The figure shows the sum of the country-level standard deviation reduction of pension expenditures before and after risk-pooling, as a % of the Eurozone aggregate GDP, using risk-unadjusted premium (red), CAPM risk-adjusted premiums (green), and proportional risk-adjusted premiums (blue). All quantities rise as the amount of mortality risk increases, but then fall as the cross-country correlations rise. All measures peak around the year 2050.

**Fig. 9.** The Eurozone-wide benefits of pooling mortality risk. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

A practical, but significant, concern is that contracts written 50 years in advance are extremely unlikely to be optimal *ex post*: the level of uncertainty is simply too large. One option would be to provide risk-pooling on a closed-system basis by pooling the mortality risks of each cohort separately, at around age 60, but only for pension payments after the age of around 70, when most uncertainty about retirement behavior has been resolved. The terms of insurance can be updated for each cohort as new information about the development of lifespan becomes available, and as pension systems are parametrically reformed. Because the benefits of risk-pooling are greatest around 30 years out, according to our measures, this would capture most of the benefits of mortality risk pooling without risking contracts that, in the long term, prove to be seriously suboptimal and therefore unenforceable in practice.

In summary, our work therefore suggests that a proposal worth exploring would be to pool the mortality risk of each cohort separately in respect of payments after around age 70, and to enter into these contracts when the cohort reaches around age 60. To control moral hazard, the pension amounts based on rules and projections in place at the time the cohort reaches age 60, rather than the pension amounts actually paid, should be pooled. This approach leaves risks at the margin to be borne by each country rather than by the pool, preserving first-best incentives for pension reform. Modern computing technology can allow each individual to be included in the pool with a full set of projected pension amounts, similar to the way that individual mortgages are included in securitizations.[17]

While we have confidence that our main results (that the benefits of pooling are macro-economically significant, and that they peak somewhere around a thirty-year horizon) are highly robust, our precise numerical estimates are dependent on a number of simplifying assumptions. Further research relaxing these assumptions will be needed to refine them. First, and most importantly, we ignore systematic differences in mortality and pension entitlements between males and females, and across different socio-economic groups. This may be highly important, but systematic mortality data (by socio-economic status), and pension projection data (by sex and socio-economic status) across the Eurozone to allow us to tease out this effect simply does not exist.

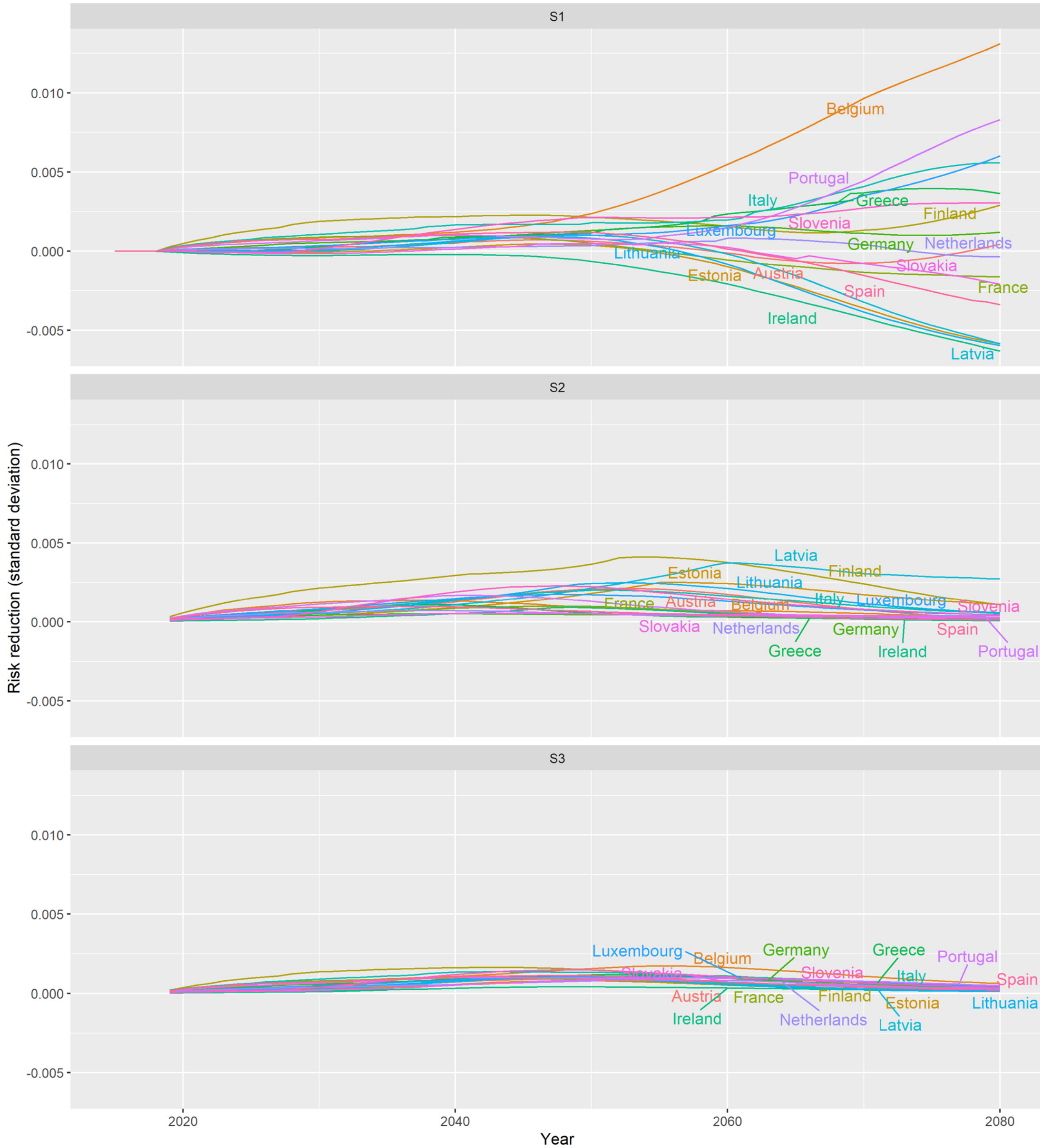
Second, we use a highly stylized pension model based on replicating AWG projections using estimated productivity and pension-increase-adjusted average pension amounts in each future year. As we explain, a fully-calibrated pension model would be extremely complicated to generate, and well beyond the scope of this paper.

Third, we use estimates of changes in labor productivity based on Eurostat estimates of accrued pension liabilities to project changes in the relative GDP of the different countries. Some assumption is necessary to ensure consistency between the Eurostat (2015) estimates of accrued pension liabilities and the AWG projections (given as a percentage of GDP).

Fourth, our mortality model is only a single model out of many possible choices, and it makes important limiting assumptions, as is the case with all mortality models. In particular, it assumes that long-latency causes of death, modeled here by cohort effects, will dominate short-latency ones (as was the case in recent years, at least until COVID-19), that there will be long-run coherence in the sense of Li and Lee (2005) in mortality rates across the Eurozone, and that there is no regional or sex-dependent clustering.

Finally, when projecting mortality rates forward, we assume no correlation in period effects between countries, despite the fact that McCarthy and Wang (2021) found evidence of these along both geographical and cultural lines in their analysis of world mortality rates using a similar model to the one used here. As the

[17] Under this approach, when the cohort turns 60, projected pensions for each individual member of the cohort in each future year would be calculated and stored. Pension expenditures pooled are then these pension amounts adjusted for the *actual* mortality of each cohort in each future year, rather than actual pension expenditures in each future year. Individual countries would be responsible for the cost of any deviations between the pensions actually paid to surviving individuals and the pension amounts projected when those individuals entered the risk pool at age 60.




Note: The top panel shows the reduction in the standard deviation of the pension expenditure in each country, using risk-unadjusted premiums. The middle panel shows the country-specific benefits under the CAPM adjustment, and the bottom panel shows the reduction in the standard deviation of the pension expenditure under the proportional risk adjustment of premiums. Smaller countries appear to benefit more under the CAPM than under the proportional scheme.

**Fig. 10.** Country-specific benefits of pooling mortality risk. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

benefits of mortality pooling are so small at short time horizons anyway, this assumption is, in our view, likely to be economically immaterial over the horizon length examined by our paper.

As a final note, it is important to emphasize that mortality is a relatively minor risk in SS systems, at least as far as open-system expenditures are concerned. As can be seen in the pension fiches of the AWG (2018), most of the long-term risk in open SS systems lies in fertility, labor force participation and wages. Extending risk-pooling to these risks would therefore bring much more significant benefits than we have calculated here. In fact, more broadly, some form of pooling GDP risks might represent an alternative risk-transfer mechanism for the EU that does not rely on centralized fiscal arrangements. Both of these will be the object of future research.

## Declaration of competing interest

There is no competing interest.

## Appendix A

We assume that the VAR is generated by a process with true parameter matrix $A$. We obtain an estimate of $A$, which we call $\hat{A}$. Regard $A$ as the true but unknown value of the parameter matrix and $\hat{A}$ as the observed value (different because of, for example, sampling error).

We want to calculate the sampling distribution of the spectral radius of $\hat{A}$, called $\rho(\hat{A})$. The VAR will be stable iff $\rho(A) < 1$. We wish to test the hypothesis that the data are generated by an underlying VAR is that is stable, even if the particular estimate $\hat{A}$ would imply instability.

We therefore approximate the probability that the true (unobserved) spectral radius is less than 1 using simulation.

*Algorithm*

1. To speed convergence of the spectral radius, we first normalize each time series to have a mean of 1. Since this is simply multiplying by a constant, it has no effect on the stability of the underlying relationship between the two variables.
2. We then estimate the VAR, and obtain an estimate of $\hat{A}$.
3. Treat $\hat{A}$ as the true value, generate 10,000 samples of the correct length, and for each sample i, fit a VAR, call the matrix of coefficients $\hat{A}_i$.
4. Calculate the spherical radius of $\hat{A}_i$ called $\rho_i$.
5. Count the number of occurrences where $\rho_i < 1$.

## Appendix B

This appendix contains summaries of the national pension systems discussed in the article. Each description is an abridgment of the 2018 pension fiche of the relevant country, prepared for the AWG of the European Commission. For more details, please see those fiches, which can be obtained at https://ec.europa.eu/info/publications/economy-finance/2018-ageing-report-economic-and-budgetary-projections-eu-member-states-2016-2070_en. Projections for all countries include old-age and survivor pensions, as well as disability pensions, provided these are covered by the public sector system. The projections include all reforms legislated before 2018, and make the assumption that reforms will be implemented as provided for in their enabling legislation.

Most Eurozone countries run earnings-based or flat-rate defined benefit (DB) systems. In these systems, pension amounts are calculated using a formula that depends on earnings, the number of contributions and the age of retirement. Pension increases are specified independently. System parameters such as retirement ages, pension increases, and initial pension amounts are often adjusted on an ad-hoc basis to control system cost, although adjustments are usually phased in over many years. Contribution rates change to track system cost, although because many systems are not fully financed by contributions and some support is obtained from general taxation, contribution rates are often set in conjunction with fiscal policy more broadly. A few countries (Luxembourg, Finland) have adopted a form of pre-funding to smooth out contribution rates. To the extent that retirement ages and pension indexation rules do not change, these systems pass the risk of changing longevity largely onto the next generation of workers in the form of higher contributions and/or taxes.

A second type of system is points-based (Germany, Lithuania, Slovakia). Here, individuals earn points by making contributions while working. The number of points earned in each period is related to the amount of contributions made relative to the contributions due from someone of average earnings (usually, a person earning average earnings earns one point per contribution period). When individuals retire, permitted at exogenously-specified ages, points are converted into a retirement income using a formula. In a pure points-based system, the conversion between points and retirement income is linear, although there are variations. The value of points changes in line with some combination of changes in wages and prices, and because, in a pure points-based system, the value of points is equal for all retirees, no matter when they retired, pension increases cannot be used to pass the costs of increased longevity onto existing pensioners without simultaneously reducing pensions for new pensioners. In a pure points-based system, contribution rates are set to balance system inflow and outgo each year, although again, there are variants. Again, to the extent that retirement ages do not change, these systems pass the risk of changes in longevity onto the subsequent generation of workers in the form of higher contributions.

The final type of system is a notional defined contribution (NDC) system (Latvia, Italy). In this system, contribution rates are fixed. Contributions are credited to a notional account, which is credited with interest in line with a set formula. When individuals retire, at a minimum age set by system rules, balances are converted into a lifetime income using an annuity factor determined using a set process. NDC systems differ from points-based systems in that the annuity factor is usually updated in line with changes in life expectancy, allowing contribution rates to stay fixed, and so the cost of increases in life expectancy are borne by the retiring generation, rather than the generations that follow them (other than changes accounted for by exogenous changes in retirement ages).

Austria

Only the first pillar of the Austrian system is covered in this paper. It is a pay-as-you-go (PAYG) defined benefit (DB) system. Separate rules apply to civil servants and private-sector workers (increasingly, public employees are being employed on private-sector contracts). The statutory retirement age is 65 years for men, and scheduled to remain so until 2070. The female retirement age is being harmonized with the male age between 2024 and 2033.

A period of 15 years contributions is required to build up entitlement to a regular old-age pension.

The assessment base is being lengthened from the best 15 years to lifetime earnings, the accrual rate has been reduced from 2% p.a. to 1.78% p.a. and annual deductions for early retirement have been increased.

Pension increases in payment are linked to consumer prices, with some small flexibility.

The contribution rate is 22.8% of gross salaries up to a cap for all workers, split between employers, employees and federal transfers (in the case of farmers, members of the liberal professions and the self-employed).

Belgium

Only the first pillar of the Belgian system is covered in this paper. It is a PAYG DB system. Separate rules apply to wage-earners, the self-employed and civil servants. The statutory retirement age is currently 65, scheduled to rise to 67 in 2030. Each scheme has different accrual rates, maximum and minimum pensions and indexation provisions, with the civil service scheme being significantly more generous.

For wage-earners and the self-employed, pensions in payment are indexed to CPI with additional increases partially adjusting to changes in living standards according to the ‘generational pact’, agreed between the social partners. All social security funds for workers (unemployment, disability, retirement) are subject to ‘global management’, meaning that there is a single contribution rate paid for all benefits and the schemes are self-financing. Civil servant pensions are broadly linked to increases in the wage base of civil servants, with all expenses forming part of the federal budget.

Estonia

Only the first pillar of the Estonian system is covered in this paper. It is a PAYG DB scheme, but has been significantly reformed following the introduction of a mandatory, funded second pillar. Generosity is therefore scheduled to fall dramatically over the next fifty years.

The pension amount is determined by a base amount, plus an amount representing accruals for pre-1999 service, and an amount representing accruals for post-1999 service. Both are essentially points-based systems, with different methods used to determine the points for pre- and post-1999 service, and different methods of indexation for the different service.

The indexation formula is complex, and depends on a mix of the change in social taxes collected and inflation. The base amount is indexed by a greater amount than the service component. The portion of pensions in respect of pre-1999 service do not currently appear to be increased. Every five years the government is required to analyze the impact of indexation on fiscal and social sustainability and suggest possible changes to Parliament.

Retirement ages are scheduled to increase to 65 for both men and women by 2030.

Social security contributions are fixed at 16% of gross pay for members of the mandatory second pillar and 20% for those not.

Finland

The Finnish public system consists of three components, all of which are included in this analysis. The main component is an earnings-related pension, which is a partly-funded (but largely PAYG) DB system. Around one quarter of this system is funded, as a means of smoothing out the effects of the baby boom generation. Because this paper focuses on expenditure, the funding arrangements are not relevant here. To top up the pensions of those with low lifetime earnings, a national pension and a guarantee pension are added to the earnings-related pension.

From 2009, the earnings-related pension amount calculated at retirement is multiplied by a factor based on life expectancy that ensures that all increases in the capital value of the pension as a result of changes in life expectancy are eliminated.

Pensions in payment are indexed at 80% of price inflation and 20% of wage inflation.

Until 2030, the retirement age will be increased until it reaches 65 for both men and women. After then, the retirement age will be changed to ensure that the fraction of adult life spent in work is unchanged from its value in 2025.

France

France has a complex pension system comprised of a number of schemes, all with different rules and entitlements. However, all schemes operate on a PAYG DB basis, and are deemed to be public-sector schemes. All are therefore included in this analysis. Various reforms, enacted in 1993, 2003, 2008, 2010 and 2014 have served to limit the fiscal impact of population ageing and simplify the system. A mooted 2020 reform aims to harmonize benefits and contributions across the system.

Reforms have reduced the pension level (by increasing the number of years' wages used to calculate the final pension and changing indexation of past wages to prices, from wages, and increasing the reference contribution period needed to earn a full pension, and indexed this to life expectancy gains), changed the indexation of pensions in payment (from wages to prices), added bonuses for late retirement and penalties for early retirement, and increased the retirement age at which entitlement to a full pension can be earned.

Pension funds are financed by a combination of contributions, other earmarked taxes and transfers of general revenue.

Germany

This analysis includes only the first tier of the German pension system, which is a points-based, PAYG DB system. Each year, contributors earn 'points' by paying social insurance taxes. Upon retirement, each worker earns a pension based on their accumulated points balance. Each point has a pension value that is updated using a formula.

Retirement ages have been increased from 65 to 67, to reflect rising life expectancy, and bonuses are awarded for late retirement (and penalties for early retirement), relative to this age.

The pension point value is updated annually reflecting growth in average wages, changes in the contribution base and changes in the dependency ratio of the entire system. The pension point value may never fall in nominal terms, but future changes will be reduced by 50% until the original trajectory is reached.

Contribution rates are set annually to meet expected outgo, with the proviso that a sustainability fund is held to smooth out erratic fluctuations and pro-cyclical movements in contribution rates.

Greece

Greece has two main pension schemes – a PAYG DB scheme, called the 'main' scheme, and a mandatory, PAYD Notional Defined Contribution (NDC) scheme, called the 'auxiliary' scheme. Both are included in this analysis.

As a result of the fiscal difficulties faced by Greece, there have been a series of pension overhauls. The system was comprehensively reformed in May 2016.

The main scheme has a flat-rate, national pension, and an earnings-related contributory pension. Retirement ages were set at 67, but will be adjusted with changes in life expectancy every three years from 2021 onwards. Pension indexation is equal to 50% CPI and 50% GDP growth, but is frozen until 2022. The legislation includes a sustainability clause, which says that if long-term projections show a rise in public pension expenditure of more than 2.5% of GDP relative to 2009 expenditure, relevant parameters of the pension system will be changed to bring expenditure below the targeted threshold.

The auxiliary scheme is an NDC scheme. The notional balance is increased by the growth in covered wages. At retirement, pension amounts are calculated using a joint-and-survivor annuity factor, which is updated every three years. Pensions in payment are indexed by the smaller of the notional rate of return less 1.3% and the change in prices, and can be negative. A balancing mechanism restricts indexation of pensions if the fund is in deficit.

Transition arrangements attempt to limit the impact of the changes in 2016 on the pensions of individual members by tracking a 'personal difference', and making some attempt to reduce this, focused on low-income workers.

Ireland

The Irish pension system consists of a first pillar social security scheme, and a second pillar of occupational and individual schemes. In this paper, the first pillar, and the public-sector unfunded occupational schemes (i.e. schemes for civil servants) in the second pillar are included.

The first pillar provides flat-rate payments of €238.30 per week, earned by making contributions pro-rata over a working life of 30 years. The greater the number of contributions made (but not their amount), the greater the pension will be. The qualifying age for the state pension, currently 66, will be increased to 68 by 2028. The amount of the pension is increased on an ad-hoc basis as part of annual budgetary negotiations.

There have been various reforms to the unfunded public sector pensions over the years, based on date of hire. Those employees hired after 1 January 2013, receive pensions based on career-average revalued earnings, where earnings before retirement are revalued in line with prices, a minimum pension age linked to the State Pension age, increases in payment linked to prices, and a maximum retirement age of 70.

Italy

Since 1995, Italy has run an NDC system. The contribution rate is fixed. The notional rate of return on contributions is set equal to a five-year geometric average increase in nominal GDP, and is converted to a pension at retirement using a transformation coefficient (annuity factor) calculated using mortality rates and updated every two years from 2019. Eligibility requirements (the minimum age requirement for old-age pensions and allowance, the minimum contribution requirements for early pensions and the minimum age requirement for early pensions) are indexed to changes in life expectancy at age 65. Pensions in payment are indexed to prices, with a decreasing proportion of price inflation used the larger the pension amount.

A reform introduced after the pension fiche was published reduced the retirement age from 67 to 62 (provided 38 years of contributions had been made to the system), but this change is scheduled to expire in January 2021.

Latvia

The first pillar of Latvia's retirement system is an NDC system, while the second pillar is a fully-funded privately-managed mandatory individual account DC system. Only the first pillar is included in this analysis. Contribution rates to the NDC system are fixed. The total contribution rate of 20% is split between the NDC and the DC portions of the system. Notional NDC balances are indexed relative to growth in nominal wages. Notional account balances never fall, but in the case of falls in the contribution wage sum, indexation is suspended until the original path (including negative indexation) is restored.

Accumulated capital is converted to a lifetime income at retirement using a factor based on unisex life expectancy. The minimum retirement age is increasing gradually and will reach 65 in 2025. Indexation of pensions in payment varies from year to year, based on a proportion of CPI and wage inflation. Indexation was suspended between 2009 and 2012.

Lithuania

Lithuania's first pillar has two components: a basic flat pension and an earnings-related pension, both funded on a PAYG basis. Both are included in this projection. The earnings-related pension switched from a DB-based system to a points-based system from January 2018. Point values will be indexed to the growth of the wage sum, averaged over a seven-year period centered on the current year (projected values are used for the future) – protecting public finances against the projected fall in the working age population.

Retirement ages are increasing to 65 by 2026. Individuals qualify to retire on full pensions once they have worked for 30 years, increasing to 35 by 2027. Adjustments are made for those who retire early or late.

Luxembourg

Luxembourg's first pillar, included here, is a PAYG DB system. The formula is complex, consisting of four separate pieces.

Retirement age is set at 65. Pensions in payment are indexed to prices and real wage growth, although the real wage growth portion of the revaluation is reduced by 50%, or canceled, when the system's benefit payments exceed its revenue. Currently, the system is running a surplus equal to around 2% of the wage base, which is allocated to a reserve fund.

No reforms have been implemented.

Netherlands

The first pillar of the Dutch pension system, the public pension, provides an equal income for all pensioners at a level related to the net minimum wage. It operates on a PAYG principle. Entitlement to the pension is earned by residence in the Netherlands, with 50 years residence qualifying an individual for a full pension. The second pillar is a mandatory, privately-managed occupational system that is funded and operates on industry lines. On average, the first and second pillars are roughly equal in size, but only the first pillar is included in this analysis.

The amount of the first pillar is indexed in line with changes in the net minimum wage.

The eligibility age is rising, and will reach 67 in 2021. From that point onward, it will be indexed in 3-month increments to changes in life expectancy of 65 years old, as projected by Statistics Netherlands, and is scheduled to reach 72 years and 6 months by 2070.

Portugal

The Portuguese system consists of a first pillar, and a private occupational system that may be substitutive or complementary to the first pillar. Civil servants were subject to different rules until 2005, but their benefits are gradually being harmonized to the general social security rules. In addition, the first pillar of the banking system – the most substantive substitutive occupational system – was shifted back into the public system in full in 2011. Only the first pillar and the civil servants scheme are included in this analysis.

The pension amount depends on reference earnings (revalued using price inflation), an accrual factor, which varies between 30% and 92% depending on earnings and length of service, and a sustainability factor applied to early retirement pensions (equal to the ratio of life expectancy at 65 in the year of retirement and life expectancy at 65 in 2000).

The retirement age is rising by two-thirds of the evolution of life expectancy at 65, and is scheduled to reach 69 years and 4 months by 2070. Pensions in payment are indexed by real GDP growth and changes in consumer prices, although the amount of the indexation depends on the amount of the pension (with smaller pensions being indexed more favorably). Indexation was suspended between 2010 and 2016.

Slovakia

The first pillar of the Slovakian pension system is a points-based, PAYG DB system. The second pillar, a fully-funded voluntary DC system, is not covered in this paper. The retirement age was gradually increased to 62 by 2017. From that point onwards, the retirement age is indexed to the year-on-year difference of the 5-year moving average of unisex life expectancy.

Points are allocated using a redistributive formula of the ratio of the individual's wages to the average covered wages in each year. Taxes are paid on wages up to seven times this average (this number has been increased several times, and was initially 3), but only 2.3 points can be granted to any individual in any year. Point values are indexed to the average wage.

Pensions in payment have historically been indexed to a combination of prices and wages. From 2018 onwards, pensions will be indexed to price inflation (measured for pensioners) with a minimum level of indexation.

Individuals who participate in the second pillar have a portion of their social security taxes redirected towards the funded pension system, with a proportional reduction of their benefit from the first pillar.

A separate arrangement is in place for members of the armed forces. It is also run on a PAYG DB basis, and is included in this paper.

Slovenia

The first pillar of the Slovenian system is a PAYG DB system. The statutory retirement age is 65 and scheduled to remain so, although the minimum age at which a pension can first be claimed depends on both age and the number of years of contributions and has been steadily increasing until 2020 when full equality between men and women will be reached. From this point onwards, these parameters are fixed.

The wage used to calculate pensions are the most favorable 24-year continuous period of membership of the plan, with a minimum of 76.5% of average gross wages, and a maximum of four times the minimum. The accrual rate is 1.25% per year after a 15-year period has been met (a slightly higher accrual rate applies for the first 15 years). Reductions and bonuses are applied for early and late retirements, respectively.

Pensions in payment are indexed to 60% of changes in wages and 40% changes in prices, although different indexation arrangements have been made at various points in the past.

A funded DC occupational pension system for those people in demanding jobs covers their pension between the time they are allowed to retire and when they first qualify for the pillar I pension. This is not included in these projections (although some small payments in respect of a minimum guaranteed pension in this system are).

Spain

The public pension system of Spain has two components – an earnings-related PAYG DB scheme and a non-earnings-related basic scheme, also PAYG. Both are included in this paper.

The statutory retirement age is gradually being increased from 65 to 67, which will be reached by 2030. Pension amounts are calculated using the average of 300 months' salary prior to retirement, with indexation of earlier months using the CPI. Bonuses for working past the full retirement age, and penalties for retiring before the full retirement age, have been introduced.

Indexation of pensions has been changed from prices to an Index for Pension Revaluation, which only allows pensions to be increased when there is no structural deficit in the pension system. Projections indicate that the IPR will remain below the legislated minimum (0.25% p.a.) until 2058. Initial pension amounts will also be adjusted from 2019 using a sustainability factor, which takes account of (smoothed) changes in life expectancy at 67. The sustainability factor ensures that the capital value of pensions at 67 remains unchanged despite changes in life expectancy.

## Appendix C

This appendix illustrates how premiums can be risk-adjusted to ensure that each country enjoys the same absolute reduction in the standard deviation of pension expenditure (as a % of GDP) as a result of participating in the risk pool. This allocation of premium is not always feasible, however, especially in earlier years where some countries have very small standard deviations. We therefore relegate it to an appendix.

We first calculate

$$P_{j,t}^{S3} = P_{j,t} + \beta_{j,t}^{E}(P_{g,t}^{*} - \bar{P}_{g,t}^{*}) \times GDP_{j,t} - (P_{j,t} - \bar{P}_{j,t})$$
$$= \bar{P}_{j,t} + \beta_{j,t}^{E}(P_{g,t}^{*} - \bar{P}_{g,t}^{*}) \times GDP_{j,t},$$

and as a fraction of GDP, as:

$$P_{j,t}^{*,S3} = \bar{P}_{j,t}^{*} + \beta_{j,t}^{E}(P_{g,t}^{*} - \bar{P}_{g,t}^{*}).$$

We then reduce the risk-equalization conditions and the self-financing condition to a set of $N$ linear equations in $N$ unknowns as follows:

$$sd(P_{j,t}^{*,S3}) - sd(P_{j,t}^{*})$$
$$= \left|\beta_{j,t}^{E}\right| sd(P_{g,t}^{*}) - sd(P_{j,t}^{*})$$
$$= \left|\beta_{j+1,t}^{E}\right| sd(P_{g,t}^{*}) - sd(P_{j+1,t}^{*})\ \forall j = 1...N-1$$
$$\sum_{j} \beta_{j,t}^{E} GDP_{j,t} = GDP_{g,t}$$

We first assume that all $\beta_{i,t}^{E}$'s are positive.

Writing the vector

$$\boldsymbol{\beta}_t^E = \{\beta_{1,t}^E, \beta_{2,t}^E, ..., \beta_{N,t}^E\}',$$

the vector

$$\boldsymbol{\gamma}_t^E = \{\frac{sd(P_{1,t}^*) - sd(P_{2,t}^*)}{sd(P_{g,t}^*)}, \frac{sd(P_{2,t}^*) - sd(P_{3,t}^*)}{sd(P_{g,t}^*)}, ..., \frac{sd(P_{N-1,t}^*) - sd(P_{N,t}^*)}{sd(P_{g,t}^*)}, 1\}',$$

and the $N$ x $N$ matrix $\mathbf{A}_t$ as follows:

$$\mathbf{A}_t = \begin{bmatrix} 1 & -1 & 0 & \dots \\ 0 & 1 & -1 & \dots \\ \vdots & \vdots & \vdots & \vdots \\ GDP_{1,t}/GDP_{g,t} & GDP_{2,t}/GDP_{g,t} & GDP_{3,t}/GDP_{g,t} & \cdots \end{bmatrix},$$

the value of $\boldsymbol{\beta}_t^E$ is then given by the following equation:

$$\boldsymbol{\beta}_t^E = \mathbf{A}_t^{-1}\boldsymbol{\gamma}_t^E.$$

If all these calculated $\beta_{i,t}^{E}$'s are positive, the risk-equalization conditions will hold and the system is solved.

In our sample, a solution exists for all years after year 2035. Up to that point, at least one of the calculated beta's is less than zero and the risk-equalization conditions consequently do not hold.

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